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Classification of Conformality Models Based on Nonabelian Orbifolds

Paul H. Frampton[(a)] **and Thomas W. Kephart**[(b)]

(a)Department of Physics and Astronomy,

University of North Carolina, Chapel Hill, NC 27599.

(b)Department of Physics and Astronomy,

Vanderbilt University, Nashville, TN 37325.

Abstract

A systematic analysis is presented of compactifications of the IIB superstring on $AdS_5 \times S^5/\Gamma$ where Γ is a non-abelian discrete group. Every possible Γ with order $g \leq 31$ is considered. There exist 45 such groups but a majority cannot yield chiral fermions due to a certain theorem that is proved. The lowest order to embrace the nonSUSY standard $SU(3) \times SU(2) \times U(1)$ model with three chiral families is $\Gamma = D_4 \times Z_3$, with $g = 24$; this is the only successful model found in the search. The consequent uniqueness of the successful model arises primarily from the scalar sector, prescribed by the construction, being

sufficient to allow the correct symmetry breakdown.

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I. INTRODUCTION

In particle phenomenology, the impressive success of the standard theory based on $SU(3) \times SU(2) \times U(1)$ has naturally led to the question of how to extend the theory to higher energies? One is necessarily led by weaknesses and incompleteness in the standard theory. If one extrapolates the standard theory as it stands one finds (approximate) unification of the gauge couplings at $\sim 10^{16}$ GeV. But then there is the *hierarchy* problem of how to explain the occurrence of the tiny dimensionless ratio $\sim 10^{-14}$ of the weak scale to the unification scale. Inclusion of gravity leads to a *super-hierarchy* problem of the ratio of the weak scale to the Planck scale, $\sim 10^{19}$ GeV, an even tinier $\sim 10^{-17}$ dimensionless ratios. Although this is obviously a very important problem about which conformality by itself is not informative, we shall discuss first the hierarchy rather than the super-hierarchy.

There are four well-defined approaches to the hierarchy problem:

- 1. Supersymmetry
- 2. Technicolor.
- 3. Extra dimensions.
- 4. Conformality.

Supersymmetry has the advantage of rendering the hierarchy technically natural, that once the hierarchy is put in to the lagrangian it need not be retuned in perturbation theory. Supersymmetry predicts superpartners of all the known particles and these are predicted to be at or below a TeV scale if supersymmetry is related to the electroweak breaking. Inclusion of such hypothetical states improves the gauge coupling unification. On the negative side, supersymmetry does not explain the origin of the hierarchy.

Technicolor postulates that the Higgs boson is a composite of fermion-antifermion bound by a new (technicolor) strong dynamics at or below the TeV scale. This obviates the hierarchy problem. On the minus side, no convincing simple model of technicolor has been found.

Extra dimensions can have a range as large as $1(\mathrm{TeV})^{-1}$ and the gauge coupling unification can happen quite differently than in only four spacetime dimensions. This replaces the hierarchy problem with a different fine-tuning question of why the extra dimension is restricted to a distance corresponding to the weak interaction scale. There is also a potentially serious problem with the proton lifetime.

Conformality is inspired by superstring duality and assumes that the particle spectrum of the standard model is enriched such that there is a conformal fixed point of the renormalization group at the TeV scale. Above this scale the coupling do not run so the hierarchy is nullified.

Conformality is the approach followed in this paper. We shall systematicaly analyse the compactification of the IIB superstring on $AdS_5 \times S^5/\Gamma$ where Γ is a discrete non-abelian group.

The duality between weak and strong coupling field theories and then between all the different superstring theories has led to a revolution in our understanding of strings. Equally profound, is the AdS/CFT duality which is the subject of the present article. This AdS/CFT duality is between string theory compactified on Anti-de-Sitter space and Conformal Field Theory.

Until very recently, the possibility of testing string theory seemed at best remote. The advent of AdS/CFTs and large-scale string compactification suggest this point of view may be too pessimistic, since both could lead to $\sim 100TeV$ evidence for strings. With this thought in mind, we are encouraged to build AdS/CFT models with realistic fermionic structure, and reduce to the standard model below $\sim 1TeV$.

Using AdS/CFT duality, one arrives at a class of gauge field theories of special recent interest. The simplest compactification of a ten-dimensional superstring on a product of an AdS space with a five-dimensional spherical manifold leads to an $\mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory, well known to be conformally invariant [1]. By replacing the manifold S^5 by an orbifold S^5/Γ one arrives at less supersymmetries corresponding to $\mathcal{N} = 2,\ 1$ or 0 depending [2] on whether: (i) $\Gamma \subset SU(2)$, (ii) $\Gamma \subset SU(3)$ but $\Gamma \not\subset SU(2)$, or (iii) $\Gamma \subset SU(4)$ but $\Gamma \not\subset SU(3)$ respectively, where Γ is in all cases a subgroup of $SU(4) \sim SO(6)$, the isometry of the S^5 manifold.

It was conjectured in [3] that such $SU(N)$ gauge theories are conformal in the $N \to \infty$ limit. In [4] it was conjectured that at least a subset of the resultant nonsupersymmetric $\mathcal{N} = 0$ theories are conformal even for finite N and that one of this subset provides the right extension of the standard model. Some first steps to check this idea were made in [5]. Model-building based on abelian Γ was studied further in [6–8], arriving in [8] at an $SU(3)^7$ model based on $\Gamma = Z_7$ which has three families of chiral fermions, a correct value for $\sin^2\theta$ and a conformal scale ~ 10 TeV.

The case of non-abelian orbifolds bases on non-abelian Γ has not previously been studied, partially due to the fact that it is apparently somewhat more mathematically sophisticated. However, we shall show here that it can be handled equally as systematically as the abelian case and leads to richer structures and interesting results.

In such constructions, the cancellation of chiral anomalies in the four-dimensional theory, as is necessary in extension of the standard model (*e.g.* [9,10]), follows from the fact that the progenitor ten-dimensional superstring theory has cancelling hexagon anomaly [11].

We consider all non-abelian discrete groups of order $g < 32$. These are described in detail in [12,15]. There are exactly 45 such non-abelian groups. Because the gauge group arrived at by this construction [6] is $\otimes_i SU(Nd_i)$ where d_i are the dimensions of the irreducible representations of Γ, one can expect to arrive at models such as the Pati-Salam $SU(4) \times SU(2) \times SU(2)$ type [16] by choosing $N = 2$ and combining two singlets and a doublet in the $\mathbf{4}$ of $SU(4)$. Indeed we shall show that such an accommodation of the standard model is possible by using a non-abelian Γ.

The procedures for building a model within such a conformality approach are: (1) Choose Γ; (2) Choose a proper embedding $\Gamma \subset SU(4)$ by assigning the components of the $\mathbf{4}$ of $SU(4)$ to irreps of Γ, while at the same time ensuring that the $\mathbf{6}$ of $SU(4)$ is real; (3) Choose N, in the gauge group $\otimes_i SU(Nd_i)$. (4) Analyse the patterns of spontaneous symmetry breaking.

In the present study we shall most often choose $N = 2$ and aim at the gauge group $SU(4) \times SU(2) \times SU(2)$. To obtain chiral fermions, it is necessary [6] that the $\mathbf{4}$ of $SU(4)$ be complex $\mathbf{4} \neq \mathbf{4}^*$. Actually this condition is not quite sufficient to ensure chirality in the present case because of the pseudoreality of $SU(2)$. We must ensure that the $\mathbf{4}$ is not just pseudoreal.

This last condition means that many of our 45 candidates for Γ do not lead to chiral fermions. For example, $\Gamma = Q_{2n} \subset SU(2)$ has irreps of appropriate dimensionalities for our purpose but with $N = 2$ it will not sustain chiral fermions under $SU(4) \times SU(2) \times SU(2)$ because these irreps are all, like $SU(2)$, pseudoreal.[1] Applying the rule that $\mathbf{4}$ must be

[1]Note that were we using $N \geq 3$ then a pseudoreal $\mathbf{4}$ would give chiral fermions.

neither real nor pseudoreal leaves a total of only 19 possible non-abelian discrete groups of order $g \leq 31$. The smallest group which avoids pseudoreality has order $g = 16$ but gives only two families. The technical details of our systematic search will be given in Sections V and VI. The simplest interesting non-abelian case which has $g = 24$ and gives three chiral families in a Pati-Salam-type model [16].

Before proceeding to details, it is worth reminding the reader that the Conformal Field Theory (CFT) that it exemplifies should be free of all divergences, even logarithmic ones, if the conformality conjecture is correct, and be completely finite. Further the theory is originating from a superstring theory in a higher-dimension (ten) and contains gravity [17–19] by compactification of the higher-dimensional graviton already contained in that superstring theory. In the CFT as we derive it, gravity is absent because we have not kept these graviton modes - of course, their influence on high-energy physics experiments is generally completely negligible unless the compactification scale is "large" [20]; here we shall neglect the effects of gravity.

It is worthwhile noting the degree of constraint imposed on the symmetry and particle content of a model as the number of irreps N_R of the discrete group Γ associated with the choice of orbifold changes. The number of gauge groups grows linearly in N_R, the number of scalar irreps grows roughly quadratically with N_R, and the chiral fermion content is highly Γ dependent. If we require the minimal Γ that is large enough for the model generated to contain the fermions of the standard model and have sufficient scalars to break the symmetry to that of the standard model, then $\Gamma = Q \times Z_3$ appears to be that minimal choice [21].

Although a decade ago the chances of testing string theory seemed at best remote, recent progress has given us hope that such tests may indeed be possible in AdS/CFTs. The model provided here demonstrates the standard model can be accommodated in these theories and suggests the possibility of a rich spectrum of new physics just around the TeV corner.

II. NON-ABELIAN GROUPS WITH ORDER $G \leq 31$

¿From any good textbook on finite groups [12] we may find a tabulation of the number of finite groups as a function of the order g, the number of elements in the group. Up to order 31 there is a total of 93 different finite groups of which slightly over one half (48) are abelian.

Amongst finite groups, the non-abelian examples have the advantage of non-singlet irreducible representations which can be used to inter-relate families. Which such group to select is based on simplicity: the minimum order and most economical use of representations [13].

Let us first dispense with the abelian groups. These are all made up from the basic unit Z_p, the order p group formed from the p^{th} roots of unity. It is important to note that the the product Z_pZ_q is identical to Z_{pq} if and only if p and q have no common prime factor.

If we write the prime factorization of g as:

$$g = \prod_i p_i^{k_i} \tag{1}$$

where the product is over primes, it follows that the number $N_a(g)$ of inequivalent abelian groups of order g is given by:

$$N_a(g) = \prod_{k_i} P(k_i) \tag{2}$$

where $P(x)$ is the number of unordered partitions of x. For example, for order $g = 144 = 2^4 3^2$ the value would be $N_a(144) = P(4)P(2) = 5 \times 2 = 10$. For $g \leq 31$ it is simple to evaluate $N_a(g)$ by inspection. $N_a(g) = 1$ unless g contains a nontrivial power ($k_i \geq 2$) of a prime. These exceptions are: $N_a(g = 4, 9, 12, 18, 20, 25, 28) = 2$; $N_a(8, 24, 27) = 3$; and $N_a(16) = 5$. This confirms that:

$$\sum_{g=1}^{31} N_a(g) = 48 \tag{3}$$

We do not consider the abelian cases further in this paper.

Of the nonabelian finite groups, the best known are perhaps the permutation groups S_N (with $N \geq 3$) of order $N!$ The smallest non-abelian finite group is S_3 ($\equiv D_3$), the symmetry of an equilateral triangle with respect to all rotations in a three dimensional sense. This group initiates two infinite series, the S_N and the D_N. Both have elementary geometrical significance since the symmetric permutation group S_N is the symmetry of the N-plex in N dimensions while the dihedral group D_N is the symmetry of the planar N-agon in 3 dimensions. As a family symmetry, the S_N series becomes uninteresting rapidly as the order and the dimensions of the represention increase. Only S_3 and S_4 are of any interest as symmetries associated with the particle spectrum [14], also the order (number of elements) of the S_N groups grow factorially with N. The order of the dihedral groups increase only linearly with N and their irreducible representations are all one- and two- dimensional. This is reminiscent of the representations of the electroweak $SU(2)_L$ used in Nature.

Each D_N is a subgroup of $O(3)$ and has a counterpart double dihedral (also known as dicyclic) group Q_{2N}, of order $4N$, which is a subgroup of the double covering $SU(2)$ of $O(3)$.

With only the use of D_N, Q_{2N}, S_N and the tetrahedral group T (of order 12, the even permutations subgroup of S_4) we find 32 of the 45 nonabelian groups up to order 31, either as simple groups or as products of simple nonabelian groups with abelian groups: (Note that $D_6 \simeq Z_2 \times D_3, D_{10} \simeq Z_2 \times D_5$ and $D_{14} \simeq Z_2 \times D_7$) Some of these groups are firmiliar from crystalography and chemistry, but the nonabelian groups that do not embed in in $SU(2)$ are less to have seen wide usage.

g	
6	$D_3 \equiv S_3$
8	$D_4, Q = Q_4$
10	D_5
12	D_6, Q_6, T
14	D_7
16	$D_8, Q_8, Z_2 \times D_4, Z_2 \times Q$
18	$D_9, Z_3 \times D_3$
20	D_{10}, Q_{10}
22	D_{11}
24	$D_{12}, Q_{12}, Z_2 \times D_6, Z_2 \times Q_6, Z_2 \times T,$
	$Z_3 \times D_4, Z_3 \times Q, Z_4 \times D_3, S_4$
26	D_{13}
28	D_{14}, Q_{14}
30	$D_{15}, D_5 \times Z_3, D_3 \times Z_5$

There remain thirteen others formed by twisted products of abelian factors. Only certain such twistings are permissable, namely (completing all $g \leq 31$)

g	
16	$Z_2 \tilde{\times} Z_8$ (two, excluding D_8), $Z_4 \tilde{\times} Z_4, Z_2 \tilde{\times}(Z_2 \times Z_4)$ (two)
18	$Z_2 \tilde{\times}(Z_3 \times Z_3)$
20	$Z_4 \tilde{\times} Z_5$
21	$Z_3 \tilde{\times} Z_7$
24	$Z_3 \tilde{\times} Q, Z_3 \tilde{\times} Z_8, Z_3 \tilde{\times} D_4$
27	$Z_9 \tilde{\times} Z_3, Z_3 \tilde{\times}(Z_3 \times Z_3)$

It can be shown that these thirteen exhaust the classification of *all* inequivalent finite groups up to order thirty-one [12].

Of the 45 nonabelian groups, the dihedrals (D_N) and double dihedrals (Q_{2N}), of order 2N and 4N respectively, form the simplest sequences. In particular, they fall into subgroups of $O(3)$ and $SU(2)$ respectively, the two simplest nonabelian continuous groups.

For the D_N and Q_{2N}, the multiplication tables, as derivable from the character tables, are simple to express in general. D_N, for odd N, has two singlet representations $1, 1^{'}$ and $m = (N-1)/2$ doublets $2_{(j)}$ $(1 \leq j \leq m)$. The multiplication rules are:

$$1^{'} \times 1^{'} = 1; \quad 1^{'} \times 2_{(j)} = 2_{(j)} \tag{4}$$

$$2_{(i)} \times 2_{(j)} = \delta_{ij}(1 + 1^{'}) + 2_{(min[i+j, N-i-j])} + (1 - \delta_{ij})2_{(|i-j|)} \tag{5}$$

For even N, D_N has four singlets $1, 1^{'}, 1^{''}, 1^{'''}$ and $(m-1)$ doublets $2_{(j)}$ $(1 \leq j \leq m - 1)$where $m = N/2$ with multiplication rules:

$$1^{'} \times 1^{'} = 1^{''} \times 1^{''} = 1^{'''} \times 1^{'''} = 1 \tag{6}$$

$$1^{'} \times 1^{''} = 1^{'''}; 1^{''} \times 1^{'''} = 1^{'}; 1^{'''} \times 1^{'} = 1^{''} \tag{7}$$

$$1^{'} \times 2_{(j)} = 2_{(j)} \tag{8}$$

$$1^{''} \times 2_{(j)} = 1^{'''} \times 2_{(j)} = 2_{(m-j)} \tag{9}$$

$$2_{(j)} \times 2_{(k)} = 2_{|j-k|} + 2_{(min[j+k,N-j-k])} \tag{10}$$

(if $k \neq j, (m-j)$)

$$2_{(j)} \times 2_{(j)} = 2_{(min[2j,N-2j])} + 1 + 1^{'} \tag{11}$$

(if $j \neq m/2$)

$$2_{(j)} \times 2_{(m-j)} = 2_{|m-2j|} + 1^{''} + 1^{'''} \tag{12}$$

(if $j \neq m/2$)

$$2_{m/2} \times 2_{m/2} = 1 + 1^{'} + 1^{''} + 1^{'''} \tag{13}$$

This last is possible only if m is even and hence if N is divisible by *four*.

For Q_{2N}, there are four singlets 1, $1^{'}$,$1^{''}$, $1^{'''}$ and $(N-1)$ doublets $2_{(j)}$ $(1 \leq j \leq (N-1))$. The singlets have the multiplication rules:

$$1 \times 1 = 1^{'} \times 1^{'} = 1 \tag{14}$$

$$1^{''} \times 1^{''} = 1^{'''} \times 1^{'''} = 1^{'} \tag{15}$$

$$1^{'} \times 1^{''} = 1^{'''}; 1^{'''} \times 1^{'} = 1^{''} \tag{16}$$

for $N = (2k+1)$ but are identical to those for D_N when N = 2k.

The products involving the $2_{(j)}$ are identical to those given for D_N (N even) above.

This completes the multiplication rules for 19 of the 45 groups. As they are not available in the literature, and somewhat tedious to work out, we have provided the complete multiplication tables for all the nonabelian groups with order $g \leq 31$ in the Appendix.

III. MATHEMATICAL THEOREM

Theorem: A Pseudoreal 4 of $SU(4)$ Cannot Yield Chiral Fermions.

In [6] it was proved that if the embedding in $SU(4)$ is such that the $\mathbf{4}$ is real: $\mathbf{4} = \mathbf{4}^*$, then the resultant fermions are always non-chiral. It was implied there that the converse holds, that if $\mathbf{4}$ is complex, $\mathbf{4} = \mathbf{4}^*$, then the resulting fermions are necessarily chiral. Actually for $\Gamma \subset SU(2)$ one encounters the intermediate possibility that the $\mathbf{4}$ is *pseudoreal*. In the present section we shall show that if $\mathbf{4}$ is pseudoreal then the resultant fermions are necessarily non-chiral. The converse now holds: if the $\mathbf{4}$ is neither real nor pseudoreal then the resutant fermions are chiral.

For $\Gamma \subset SU(2)$ it is important that the embedding be consistent with the chain $\Gamma \subset SU(2) \subset SU(4)$ otherwise the embedding is not a consistent one. One way to see the inconsistency is to check the reality of the $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{antisymmetric}$. If $\mathbf{6} \neq \mathbf{6}^*$ then the embedding is clearly improper. To avoid this inconsistency it is sufficient to include in the $\mathbf{4}$ of $SU(4)$ only complete irreducible representations of $SU(2)$.

An explicit example will best illustrate this propriety constraint on embeddings. Let us consider $\Gamma = Q_6$, the dicyclic group of order $g = 12$. This group has six inequivalent irreducible representations: $1, 1', 1'', 1''', 2_1, 2_2$. The 1, $1'$, 2_1 are real. The $1''$ and $1'''$ are a complex conjugate pair, The 2_2 is pseudoreal. To embed $\Gamma = Q_6 \subset SU(4)$ we must choose from the special combinations which are complete irreducible representations of $SU(2)$ namely 1, $2 = 2_2$, $3 = 1' + 2_1$ and $4 = 1'' + 1''' + 2_2$. In this way the embedding either makes the $\mathbf{4}$ of $SU(4)$ real *e.g.* $4 = 1 + 1' + 2_1$ and the theorem of [6] applies, and non-chirality results, or the $\mathbf{4}$ is pseudoreal *e.g.* $4 = 2_2 + 2_2$. In this case one can check that the embedding is consistent because $(\mathbf{4}\otimes\mathbf{4})_{antisymmetric}$ is real. But it is equally easy to check that the product of this pseudoreal $\mathbf{4}$ with the complete set of irreducible representations of Q_6 is again real and that the resultant fermions are non-chiral.

The lesson is:

To obtain chiral fermions from compactification on $AdS_5 \times S_5/\Gamma$, the embedding of Γ in $SU(4)$ must be such that the 4 of $SU(4)$ is neither real nor pseudoreal.

IV. CHIRAL FERMIONS FOR ALL NONABELIAN $G \leq 31$

Looking at the full list of non-abelian discrete groups of order $g \leq 31$ as given explicitly in [15] we see that of the 45 such groups 32 are simple groups or semi-direct products thereof; these 32 are listed in the Table on page 4691 of [15], and reproduced in section II above. The remaining 13 are formed as semi-direct product groups (SDPGs) and are listed in the Table on page 4692 of [15] and in section II. We shall follow closely this classification.

Using the pseudoreality considerations of the previous section, we can pare down the full list of 45 to only 19 which include 13 SDPGs. The lowest order nonabelian group Γ which can lead to chiral fermions is $g = 16$. The only possible orders $g \leq 31$ are the seven values:

$g = 16(5[5SDPGs]), \quad 18(2[1SDPG]), \quad 20(1[1SDPG]), \quad 21(1[1SDPG]),$

$24(6[3SDPGs]), \quad 27(2[2SDPGs]), \quad \text{and} \quad 30(2[0SDPG]).$

In parenthesis are the number of groups at order g and the number of these that are SDPGS is in square brackets; they add to (19[13 SDPGs]). We shall proceed with the analysis systematically, in progressively increasing magnitude of g.

g = 16.

The non-pseudoreal groups number five, and all are SDPGs. In the notation of Thomas and Wood [12], which we shall follow for definiteness both here and in Appendix A, they are: $16/8, 9, 10, 11, 13$. So we now treat these in the order they are enumerated by Thomas and Wood. Again, the relevant multiplication tables are collected in Appendix A.

Group 16/8; also designated $(Z_4 \times Z_2)\tilde{\times} Z_2$.

This group has eight singlets $1_1, 1_2, ..., 1_8$ and two doublets 2_1 and 2_2. In the embedding of 16/8 in $SU(4)$ we must avoid the singlet 1_1 otherwise there will be a residual supersymmetry with $\mathcal{N} \geq 1$. Consider the embedding defined by $\mathbf{4} = (2_1, 2_1)$. To find the surviving chiral fermions we need to product the $\mathbf{4}$ with all ten of the irreps of 16/8. This results in the table:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1									$\times\times$	
1_2									$\times\times$	
1_3									$\times\times$	
1_4									$\times\times$	
1_5										$\times\times$
1_6										$\times\times$
1_7										$\times\times$
1_8										$\times\times$
2_1					$\times\times$	$\times\times$	$\times\times$	$\times\times$		
2_2	$\times\times$	$\times\times$	$\times\times$	$\times\times$						

If we choose $N = 2$, the gauge group is $SU(2)^8 \times SU(4)^2$, and the entries in the table correspond to bifundamental representations of this group (e.g., the entry nearest the top right corner at the position $(1_1,\ 2_1)$ is the representation $2(2,1,1,1,1,1,1,1;\bar{4},1)$). If we identify the diagonal subgroup of the first four SU(2)s as $SU(2)_L$, of the second four as $SU(2)_R$ and of the two SU(4) as color SU(4) the result is non-chiral due to the symmetry about the main diagonal of the above table.

On the other hand, if we identify $\mathbf{4_1}$ with $\mathbf{\bar{4}_2}$ there are potentially eight chiral families:

$$8[(2,1,4) + (1,2,\bar{4})] \tag{17}$$

under $SU(2)_L \times SU(2)_R \times SU(4)$. This is the maximum total chirality for this orbifold, but as we will see in section IV, the allowed chiral at any stage is as usual determined by spontaneous symmetry breaking (SSB) generated in the scalar sector. In this section we give the maximum chirality for each orbifold, in the next section we study SSB for those models with sufficient chirality too accomodate at least three families.

Because $2_1 = 2_2^*$ form a complex conjugate pair, the embedding $\mathbf{4} = (2_1, 2_2)$ is pseudoreal $\mathbf{4} \equiv \mathbf{4}^*$ and the fermions are non-chiral as easily confirmed by the resultant table:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1									$\times$	$\times$
1_2									$\times$	$\times$
1_3									$\times$	$\times$
1_4									$\times$	$\times$
1_5									$\times$	$\times$
1_6									$\times$	$\times$
1_7									$\times$	$\times$
1_8									$\times$	$\times$
2_1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		
2_2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		

For this embedding, the result is non-chiral for either of the cases $\mathbf{4_1} \equiv \mathbf{4_2}$ or $\mathbf{4_1} \equiv \bar{\mathbf{4}}_\mathbf{2}$. (In the future, we shall not even consider such trivially real non-chiral embeddings).

Finally, for 16/8, consider the embedding $\mathbf{4} = (1_2, 1_5, 2_1)$. (In general there will be many equivalent embeddings. We will give one member of each equivalence class. Cases that are obviously nonchiral (vectorlike) will, in general, be ignored, except for a few instructive examples as order 16 and 18.) The table is now:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		$\times$			$\times$				$\times$	
1_2	$\times$					$\times$			$\times$	
1_3				$\times$			$\times$		$\times$	
1_4			$\times$					$\times$	$\times$	
1_5	$\times$						$\times$			$\times$
1_6		$\times$				$\times$				$\times$
1_7			$\times$					$\times$		$\times$
1_8				$\times$			$\times$			$\times$
2_1					$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
2_2	$\times$	$\times$	$\times$	$\times$					$\times$	$\times$

which is chiral.

These examples of embeddings for $\Gamma = 16/8$ show clearly how the number of chiral families depends critically on the choice of embedding $\Gamma \subset SU(4)$. To actual achieve a model with a viable phenomenologically, we must study the possible routes through SSB for each chiral model. This we postpone until we have found all models of potential interest.

Group 16/9; also designated $[(Z_4 \times Z_2)\tilde{\times} Z_2]'$

This group has irreps which comprise eight singlets $1_1, ..., 1_8$ and two doublets $2_1, 2_2$. With the embedding $\mathbf{4} = (2_1, 2_2)$ and using the multiplication table from Appendix A we arrive at the table of fermion bilinears:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1									$\times\times$	
1_2										$\times\times$
1_3									$\times\times$	
1_4										$\times\times$
1_5									$\times\times$	
1_6										$\times\times$
1_7									$\times\times$	
1_8										$\times\times$
2_1	$\times\times$		$\times\times$		$\times\times$		$\times\times$			
2_2		$\times\times$		$\times\times$		$\times\times$		$\times\times$		

This is non-chiral and has no families. This was the only potentially chiral embedding. In what follows, nonchiral models will not be displayed, however, as the unification scale can be rather low in AdS/CFT models, it would also be interesting to investigate vectorlike models of this class.

Group 16/10; also designated $Z_4 \tilde{\times} Z_4$

The multiplication table is identical to that for 16/9, as mentioned in Appendix A; thus the model building for 16/10 is also identical to 16/9 and merits no additional discussion.

Group 16/11; also designated $Z_8 \tilde{\times} Z_2$

Again there are eight singlets and two doublets. The singlets $1_{1,3,5,7}$ are real while the other singlets fall into two conjugate pairs: $1_2 = 1_4^*$ and $1_6 = 1_8^*$. The doublets are complex:

$2_1 = 2_2^*$.

The multiplication table in the Appendix includes the products: $1_{1,3,5,7} \times 2_{1,2} = 2_{1,2}$ and $1_{2,4,6,8} \times 2_{1,2} = 2_{2,1}$. Also $2_1 \times 2_1 = 2_2 \times 2_2 = 1_2 + 1_4 + 1_6 + 1_8$, while $2_1 \times 2_2 = 1_1 + 1_3 + 1_5 + 1_7$.

This means that there are no interesting (legitimate and chiral) embeddings of the type 1+1+2 or 2+2.

The most chiral possibility is the embedding $\mathbf{4} = (1_2, 1_2, 1_2, 1_2)$ which leads to the fermions in the following table. In this table, $(\times)^6 \equiv (\times \times \times \times \times\times)$.

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		$(\times)^6$								
1_2			$(\times)^6$							
1_3				$(\times)^6$						
1_4	$(\times)^6$									
1_5						$(\times)^6$				
1_6							$(\times)^6$			
1_7								$(\times)^6$		
1_8					$(\times)^6$					
2_1										$(\times)^6$
2_2									$(\times)^6$	

This gives rise to twelve chiral families if we identify $N = 3$ and $3_1 = 3_4 = 3_5 = 3_8$, $3_2 = 3_6$ and $3_3 = 3_7$. Under $SU(3)^3$ the chiral fermions are:

$$12[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 3, 1)] \tag{18}$$

together with real non-chiral representations. In section VI where we discuss spontaneous symmetry breaking, we will see if this type of unification is possible.

With the different embedding $\mathbf{4} = (1_2, 1_2, 1_2, 1_4)$ the model changes to a less chiral but still interesting fermion configuration:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2
1_1		$\times\times\times$		$\times$						
1_2	$\times$		$\times\times\times$							
1_3		$\times$		$\times\times\times$						
1_4	$\times\times\times$		$\times$							
1_5						$\times\times\times$		$\times$		
1_6					$\times$		$\times\times\times$			
1_7						$\times$		$\times\times\times$		
1_8					$\times\times\times$		$\times$			
2_1										$\times\times\times\times$
2_2									$\times\times\times\times$	

If we can identify $SU(3)'s$ as $3_1 \equiv 3_4 \equiv 3_5 \equiv 3_8$, $3_2 \equiv 3_6$ and $3_3 \equiv 3_7$ this embedding give just four chiral families:

$$4[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 3, 1)] \tag{19}$$

under $SU(3)^3$ together with real representations.

To check consistency, we have verified that real and legitimate embeddings for 16/11 like: $\mathbf{4} = (1_3, 1_3, 1_3, 1_3)$ and $\mathbf{4} = (2_1, 2_2)$ give no chiral fermions.

Group 16/13; also designated $[Z_8 \tilde{\times} Z_2]''$

Of the five non-pseudoreal $g = 16$ nonabelian Γ, 16/13 is unique in having only four inequivalent singlets $1_1, 1_2, 1_3, 1_4$ but three doublets $2_1, 2_2, 2_3$.

All the four singlet are real $1_i = 1_i^*$. The three doublets comprise a conjugate complex pair $2_1 = 2_3^* \neq 2_1^*$ and the real $2_2 = 2_2^*$.

With the embedding $\mathbf{4} = (1_3, 1_4, 2_1)$ the resultant model has a chiral fermion quiver corresponding to the Table:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1			$\times$	$\times$	$\times$		
1_2			$\times$	$\times$			$\times$
1_3	$\times$	$\times$					$\times$
1_4	$\times$	$\times$			$\times$		
2_1		$\times$	$\times$		$\times$	$\times$	$\times$
2_2					$\times$	$\times\times$	$\times$
2_3	$\times$			$\times$	$\times$	$\times$	$\times$

If we identify $SU(2)_L$ with the diagonal subgroup of the first and fourth $SU(2)$s, and $SU(2)_R$ with the diagonal subgroup of the 2nd and 3rd, then there are four chiral families if we embed $\mathbf{4_1} \equiv \mathbf{\bar{4}_3}$ and break $SU(4)_2$ completely.

Next, for $g = 16$, consider 16/13 with $\mathbf{4} = (2_1, 2_1)$. The table becomes:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1					$\times\times$		
1_2							$\times\times$
1_3							$\times\times$
1_4					$\times\times$		
2_1		$\times\times$	$\times\times$			$\times\times$	
2_2					$\times\times$		$\times\times$
2_3	$\times\times$			$\times\times$		$\times\times$	

With $\mathbf{4_1} \equiv \mathbf{\bar{4}_3}$ there are eight chiral families.

A similar result occurs, of course, for $\mathbf{4} = (2_3, 2_3)$. But the embedding $\mathbf{4} = (2_1, 2_3)$ is non-chiral, leading to the symmetric fermion quiver/table:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1					$\times$		$\times$
1_2					$\times$		$\times$
1_3					$\times$		$\times$
1_4					$\times$		$\times$
2_1	$\times$	$\times$	$\times$	$\times$		$\times\times$	
2_2					$\times\times$		$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$		$\times\times$	

This arrangement is manifestly non-chiral because of the symmetry of the table. Even though 2_1 and 2_3 are complex, $2_1 = 2_3^*$, so $4^* = (2_1, 2_3)^* = (2_3, 2_1)$. We can rotate this within $SU(4)$ to $4 = (2_1, 2_3)$. Therefore, the 4 is pseudoreal and the fermions are vectorlike as expected.

The embedding $\mathbf{4} = (2_2, 2_2)$ in 16/13 gives rise to the table:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1						$\times\times$	
1_2						$\times\times$	
1_3						$\times\times$	
1_4						$\times\times$	
2_1					$\times\times$		$\times\times$
2_2	$\times\times$	$\times\times$	$\times\times$	$\times\times$			
2_3					$\times\times$		$\times\times$

This embedding leads to no chirality and zero families.

Finally, the embedding $\mathbf{4} = (2_1, 2_2)$ of 16/13 leads to the intermediate situation:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1					$\times$	$\times$	
1_2						$\times$	$\times$
1_3						$\times$	$\times$
1_4					$\times$	$\times$	
2_1		$\times$	$\times$		$\times$	$\times$	$\times$
2_2	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$
2_3	$\times$			$\times$	$\times$	$\times$	$\times$

This give rise to four chiral families with the identification $\mathbf{4_1} \equiv \bar{\mathbf{4}}_\mathbf{3}$.

To summarize the "double doublet" embeddings $\mathbf{4} = (2_i, 2_j)$ of 16/13: for the equivalent embeddings (i, j) = (1, 1) or (3, 3), there are up to eight chiral families; for the other mutually equivalent cases (i, j) = (1, 2), (3, 2), (2, 3), or (2, 1) there are up to four chiral families and finally for the pseudoreal cases (i, j) = (1, 3), (3, 1) and the real case (2, 2) there are, because of the mathematical theorem (and as we have now is easyverified by direct calculation) no chiral fermions.

g = 18.

The non-pseudoreal groups number two, and one is an SDPG. In the notation of Thomas and Wood [12] they are: $18/3, 5$. So we now treat these in the order they are enumerated by Thomas and Wood.

Group 18/3; also designated $D_3 \times Z_3$

This group has irreps which fall into six singlets $1, 1^{'}, 1\alpha, 1^{'}\alpha, 1\alpha^2, 1^{'}\alpha^2$ and three doublets $2, 2\alpha, 2\alpha^2$. Using the D_3 multiplication table from appendix A the embedding $\mathbf{4} = (1\alpha, 1^{'}, 2\alpha)$ yields the table:

	1	$1^{'}$	2	1α	$1^{'}\alpha$	2α	$1\alpha^2$	$1^{'}\alpha^2$	$2\alpha^2$
1		$\times$		$\times$		$\times$			
$1^{'}$	$\times$				$\times$	$\times$			
2			$\times$	$\times$	$\times$	$\times\times$			
1α					$\times$		$\times$		$\times$
$1^{'}\alpha$				$\times$				$\times$	$\times$
2α						$\times$	$\times$	$\times$	$\times\times$
$1\alpha^2$	$\times$		$\times$					$\times$	
$1^{'}\alpha^2$		$\times$	$\times$				$\times$		
$2\alpha^2$	$\times$	$\times$	$\times\times$						$\times$

Identifying $SU(2)_{L,R}$ with the diagonal subgroups of respectively $SU(2)_3 \times SU(2)_4$ and $SU(2)_5 \times SU(2)_6$ gives rise to two chiral families when it is assumed that $SU(2)_{1,2}$ and $SU(4)_{1,2}$ are broken.

Group 18/5; also designated $(Z_3 \times Z_3)\tilde{\times} Z_2$

This group has two singlets $1, 1^{'}$ and four doublets $2_1, 2_2, 2_3, 2_4$. Using the multiplication table from Appendix A we compute the models corresponding to the three inequivalent embeddings $\mathbf{4} = (1^{'}, 1^{'}, 2_1)$, $\mathbf{4} = (2_1, 2_1)$ and $\mathbf{4} = (2_1, 2_2)$.

For $\mathbf{4} = (1^{'}, 1^{'}, 2_1)$ the table is:

	1	$1^{'}$	2_1	2_2	2_3	2_4
1		$\times\times$	$\times$			
$1^{'}$	$\times\times$		$\times$			
2_1	$\times$	$\times$	$\times\times\times$			
2_2				$\times\times$	$\times$	$\times$
2_3				$\times$	$\times\times$	$\times$
2_4				$\times$	$\times$	$\times\times$

This model is manifestly non-chiral due to the symmetry of the table.

For $\mathbf{4} = (2_1, 2_1)$ the table is:

	1	$1'$	2_1	2_2	2_3	2_4
1			$\times\times$			
$1'$			$\times\times$			
2_1	$\times\times$	$\times\times$	$\times\times$			
2_2					$\times\times$	$\times\times$
2_3				$\times\times$		$\times\times$
2_4				$\times\times$	$\times\times$	

This model is also manifestly non-chiral due to the symmetry of the table.

For $\mathbf{4} = (2_1, 2_2)$ the table is:

	1	$1'$	2_1	2_2	2_3	2_4
1			$\times$	$\times$		
$1'$			$\times$	$\times$		
2_1	$\times$	$\times$	$\times$		$\times$	$\times$
2_2	$\times$	$\times$		$\times$	$\times$	$\times$
2_3			$\times$	$\times$		$\times\times$
2_4			$\times$	$\times$	$\times\times$	

Again, this model is manifestly non-chiral. 18/5 does not lend itself to chirality.

This is easy to understand when one realizes that all of the irreducible representations of 18/5 are individually either real or pseudoreal [12] making a complex embedding of **4** impossible.

g = 20.

One non-pseudoreal group, an SDPG. In the notation of Thomas and Wood [12] it is 20/5.

Group 20/5; also designated $Z_5 \tilde{\times} Z_4$

The group has four singlets $1_1, 1_2, 1_3, 1_4$ and a 4. The singlets $1_1, 1_3$ are real and the other two form a complex conjugate pair $1_2 = 1_4^*$. The **6** which is the antisymmetric product **6** $= (4 \times 4)_a$ must be real for a legitimate embedding. The two inequivalent choices, bearing in mind the multiplication table provided in the Appendix are $\mathbf{4} = (1_2, 1_2, 1_2, 1_2)$ and $\mathbf{4} = (1_2, 1_2, 1_2, 1_4)$.

The first $\mathbf{4} = (1_2, 1_2, 1_2, 1_2)$ yields the chiral fermions in the following table:

	1_1	1_2	1_3	1_4	4
1_1		$\times\times\times\times$			
1_2			$\times\times\times\times$		
1_3				$\times\times\times\times$	
1_4	$\times\times\times\times$				
4					$\times\times\times\times\times$

Putting $N = 3$ this embedding gives four chiral families when we identify $SU(3)_3 \equiv SU(3)_4$ and drop real representations, giving:

$$4[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)] \tag{20}$$

under $SU(3) \times SU(3) \times SU(3)$. This possibility for the 20/5 nonabelian orbifold certainly merits further study.

The symmetry breaking for this model will be investigated in the subsequent section.

The second inequivalent embedding $\mathbf{4} = (1_2, 1_2, 1_2, 1_4)$ gives rise to the table:

	1_1	1_2	1_3	1_4	4
1_1		$\times\times\times$		$\times$	
1_2	$\times$		$\times\times\times$		
1_3		$\times$		$\times\times\times$	
1_4	$\times\times\times$		$\times$		
4					$\times\times\times\times\times$

Identifying $SU(3)_3 \equiv SU(3)_4$ as before for $N = 3$ this is less chiral and gives rise to just two chiral families.

$$4[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)] \tag{21}$$

under $SU(3) \times SU(3) \times SU(3)$.

g = 21.

One non-pseudoreal group, an SDPG. In the notation of Thomas and Wood [12] it is: 21/2.

Group 21/2; also designated $Z_7 \tilde{\times} Z_3$

This group has irreps which comprise three singlets $1_1, 1_2, 1_3$ and two triplets $3_1, 3_2$. With the embedding $\mathbf{4} = (1_2, 3_1)$ (recall that 1_1 must be avoided to obtain $\mathcal{N} = 0$), the resultant fermions are given by:

	1_1	1_2	1_3	3_1	3_2
1_1		$\times$		$\times$	
1_2			$\times$	$\times$	
1_3	$\times$			$\times$	
3_1				$\times\times$	$\times\times$
3_2	$\times$	$\times$	$\times$	$\times$	$\times\times$

Putting $N = 2$, the gauge group is $SU(2)^3 \times SU(6)^2$. Clearly the model is chiral as seen in the asymmetry of the table. For example, put $SU(2)_L \equiv SU(2)_1$, $SU(2)_R \equiv SU(2)_2$, break $SU(2)_3$ entirely and $\mathbf{6_1} \to \mathbf{4}, \mathbf{6_2} \to \bar{\mathbf{4}}$, to find two chiral families.

g = 24.

The non-pseudoreal groups number six, and three are SDPGs. In the notation of Thomas and Wood [12] they are: $24/7, 8, 9, 13, 14, 15$. So we now treat these in the order they are enumerated by Thomas and Wood.

Group 24/7; also designated $D_4 \times Z_3$

This has twelve singlets $1_1\alpha^i, 1_2\alpha^i, 1_3\alpha^i, 1_4\alpha^i$ (i = 0 - 2) and three doublets $2\alpha^i$ (i = 0 - 2); here $\alpha = exp(i\pi/3)$. The embedding $\mathbf{4} = (1_1\alpha, 1_2, 2\alpha)$ was studied in detail in our previous article [21] where it was shown how it can lead to precisely three chiral families in the standard model.

For completeness we include the table for the chiral fermions (it was presented in a different equivalent way in [21]):

	1_1	1_2	1_3	1_4	2	$1_1\alpha$	$1_2\alpha$	$1_3\alpha$	$1_4\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$1_3\alpha^2$	$1_4\alpha^2$	$2\alpha^2$
1_1		×				×				×					
1_2	×						×			×					
1_3				×				×		×					
1_4			×						×	×					
2					×	×	×	×	×	×					
$1_1\alpha$							×				×				×
$1_2\alpha$						×						×			×
$1_3\alpha$									×				×		×
$1_4\alpha$								×						×	×
2α										×	×	×	×	×	×
$1_1\alpha^2$	×				×							×			
$1_2\alpha^2$		×			×						×				
$1_3\alpha^2$			×		×									×	
$1_4\alpha^2$				×	×								×		
$2\alpha^2$	×	×	×	×	×										×

By identifying $SU(4)$ with the diagonal subgroup of $SU(4)_{2,3}$, breaking $SU(4)_1$ to $SU(2)_L' \times SU(2)_R'$, then identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{6,7,8}$ and $SU(2)_L'$ and $SU(2)_R$ with the diagonal subgroup of $SU(2)_{10,11,12}$ and $SU(2)_R'$ then leads to a three-family model as explained already in [21].

It is convenient to represent the chiral fermions in a quiver diagram [22] as shown in the

Figure:

Insert figure and caption here

This model is especially interesting because, uniquely among the large number of models examined in this study, the prescribed scalars are sufficient to break the gauge symmetry to that of the standard model.

Group 24/8; also designated $Q \times Z_3$

The multiplication tables of D_4 and Q and hence the multiplication tables of 24/7 and 24/8 are identical. Model building for 24/8 is therefore the same as 24/7 and merits no additional discussion.

Group 24/9; also designated $D_3 \times Z_4$

This group generates one of the richest sets of chiral model in the class of models discussed in this paper. The group has as irreps eight singlets $(1_1\alpha^j, 1_2\alpha^j)$ and four doublets $2\alpha^j$ (j = 0, 1, 2, 3), where $\alpha = exp(i\pi/4)$.

The embedding $\mathbf{4} = (1_1\alpha^{a_1}, 1_2\alpha^{a_2}, 2\alpha^{a_3})$ must satisfy $a_1 \neq 0$ (for $\mathcal{N} = 0$) and $a_1 + a_2 = -2a_3$ (mod 4) (to ensure reality of $\mathbf{6} = (\mathbf{4} \times \mathbf{4})_a$). There are several interesting possibilities including: $(1_1\alpha, 1_2\alpha, 2\alpha))$, $(1_1\alpha, 1_2\alpha^3, 2\alpha^2))$, $(1_1\alpha^2, 1_2, 2\alpha^3))$, $(1_1\alpha^2, 1_2, 2\alpha))$, and $(1_1\alpha^2, 1_2\alpha^2, 2))$. The third and fourth cases are equivalent as can be seen by letting α go to α^{-1}, and the last case has only real fermions since $\alpha^2 = -1$.

The fermions for $\mathbf{4} = (\mathbf{1_1}\alpha^{\mathbf{2}}, \mathbf{1}_2\alpha^{\mathbf{2}}, \mathbf{2})$ are vectorlike.

Moving on to 24/9 with $\mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1_2}\alpha^{\mathbf{3}}, \mathbf{2}\alpha^{\mathbf{2}})$ we find the fermions are chiral and fall into the irrep:

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1				$\times$					$\times$		$\times$	
1_2					$\times$				$\times$	$\times$		
2						$\times$	$\times$	$\times$	$\times$			$\times$
$1_1\alpha$		$\times$					$\times$					$\times$
$1_2\alpha$	$\times$							$\times$				$\times$
2α			$\times$						$\times$	$\times$	$\times$	$\times$
$1_1\alpha^2$			$\times$		$\times$					$\times$		
$1_2\alpha^2$			$\times$	$\times$							$\times$	
$2\alpha^2$	$\times$	$\times$	$\times$			$\times$						$\times$
$1_1\alpha^3$	$\times$					$\times$		$\times$				
$1_1\alpha^3$		$\times$				$\times$	$\times$					
$2\alpha^3$			$\times$	$\times$	$\times$	$\times$			$\times$			

With the embedding $\mathbf{4} = (1_1\alpha, 1_2\alpha, 2\alpha)$ the chiral fermions are:

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1				$\times$	$\times$	$\times$						
1_2				$\times$	$\times$	$\times$						
2				$\times$	$\times$	$\times\times\times$						
$1_1\alpha$							$\times$	$\times$	$\times$			
$1_2\alpha$							$\times$	$\times$	$\times$			
2α							$\times$	$\times$	$\times\times\times$			
$1_1\alpha^2$										$\times$	$\times$	$\times$
$1_2\alpha^2$										$\times$	$\times$	$\times$
$2\alpha^2$										$\times$	$\times$	$\times\times\times$
$1_1\alpha^3$	$\times$	$\times$	$\times$									
$1_2\alpha^3$	$\times$	$\times$	$\times$									
$2\alpha^3$	$\times$	$\times$	$\times\times\times$									

Identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{1,2,3,4}$, $SU(2)_R$ with the diagonal subgroup of $SU(2)_{5,6,7,8}$ and the $\mathbf{4}$ of $SU(4)$ with the $\mathbf{4}$ of $SU(4)_{2,3}$ and the $\bar{\mathbf{4}}$ of $SU(4)_{1,4}$ leads to eight chiral families.

Taking the embedding $\mathbf{4} = (1_1\alpha^2, 1_2, 2\alpha)$ gives as chiral fermions:

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1		$\times$				$\times$	$\times$					
1_2	$\times$					$\times$		$\times$				
2			$\times$	$\times$	$\times$	$\times$			$\times$			
$1_1\alpha$					$\times$				$\times$	$\times$		
$1_2\alpha$				$\times$					$\times$		$\times$	
2α						$\times$	$\times$	$\times$	$\times$			$\times$
$1_1\alpha^2$	$\times$							$\times$				$\times$
$1_2\alpha^2$		$\times$					$\times$					$\times$
$2\alpha^2$			$\times$						$\times$	$\times$	$\times$	$\times$
$1_1\alpha^3$			$\times$	$\times$							$\times$	
$1_2\alpha^3$			$\times$		$\times$					$\times$		
$2\alpha^3$	$\times$	$\times$	$\times$			$\times$						$\times$

We identify $SU(2)_L$, $SU(2)_R$ with the diagonal subgroups of $SU(2)_{1,2}$ and $SU(2)_{3,4}$, respectively and break completely $SU(2)_{5,6,7,8}$. The generalized color embedding $\mathbf{4} \equiv \mathbf{4_1} \equiv \mathbf{4_2} \equiv \mathbf{\bar{4}_3} \equiv \mathbf{\bar{4}_4}$ leads to four chiral families. This can be reduced to three families by further symmetry breaking using the same idea as in [21].

An even more interesting embedding for 24/9 is to set $\mathbf{4} = (2\alpha, 2\alpha)$ which gives a real $\mathbf{6}$ as required (since $\alpha^2 = -1$ is real). The table for fermions is:

	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1						$\times\times$						
1_2						$\times\times$						
2				$\times\times$	$\times\times$	$\times\times$						
$1_1\alpha$									$\times\times$			
$1_2\alpha$									$\times\times$			
2α							$\times\times$	$\times\times$	$\times\times$			
$1_1\alpha^2$												$\times\times$
$1_2\alpha^2$												$\times\times$
$2\alpha^2$										$\times\times$	$\times\times$	$\times\times$
$1_1\alpha^3$			$\times\times$									
$1_2\alpha^3$			$\times\times$									
$2\alpha^3$	$\times\times$	$\times\times$	$\times\times$									

Identifying $SU(2)_L$ with the diagonal subgroup of $SU(2)_{1,3,5,7}$, $SU(2)_R$ with the diagonal subgroup of $SU(2)_{2,4,6,8}$, breaking $SU(4)_{1,3}$ and keeping the unbroken $SU(4)$ which is the diagonal subgroup of $SU(4)_{2,4}$ gives rise to eight chiral families:

$$8[(2,1,\bar{4}) + (1,2,4)] \tag{22}$$

The possibility of achieving the relevant symmetry breaking will be examined below in Section V.

Group 24/13; also designated $Q\tilde{\times}Z_3$

This group has three singlets $1_1, 1_2, 1_3$, three doublets $2_1, 2_2, 2_3$ and one triplet 3. For $N = 2$ the gauge group is therefore $SU(2)^3 \times SU(4)^3 \times SU(6)$.

With the embedding $\mathbf{4} = (2_1, 2_2)$ the chiral fermions are:

	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1				$\times$	$\times$		
1_2					$\times$	$\times$	
1_3				$\times$		$\times$	
2_1	$\times$	$\times$					$\times\times$
2_2		$\times$	$\times$				$\times\times$
2_3	$\times$		$\times$				$\times\times$
3				$\times\times$	$\times\times$	$\times\times$	

If we identify $SU(2)_L \equiv SU(2)_3$, $SU(2)_R \equiv SU(2)_2$, and break $SU(2)_1$ there are two chiral families for $\mathbf{4} \equiv \mathbf{4_1} \equiv \mathbf{\bar{4}_2} \equiv \mathbf{\bar{4}_3}$.

If, instead, we embed $\mathbf{4} = (2_2, 2_3)$ the fermions fall according to the following table:

	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1					$\times$	$\times$	
1_2				$\times$		$\times$	
1_3				$\times$	$\times$		
2_1		$\times$	$\times$				$\times\times$
2_2	$\times$		$\times$				$\times\times$
2_3	$\times$	$\times$					$\times\times$
3				$\times\times$	$\times\times$	$\times\times$	

This model is manifestly non-chiral because of the total symmetry of the table.

Group 24/14; also designated $Z_8 \tilde{\times} Z_3$

There are eight singlets and four doublets, with multiplication table as in Appendix A. With the embedding $\mathbf{4} = (2_2, 2_4)$ one arrives at the fermions:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_1										$\times$		$\times$
1_2									$\times$		$\times$	
1_3										$\times$		$\times$
1_4									$\times$		$\times$	
1_5										$\times$		$\times$
1_6									$\times$		$\times$	
1_7										$\times$		$\times$
1_8									$\times$		$\times$	
2_1		$\times$		$\times$		$\times$		$\times$		$\times$		$\times$
2_2	$\times$		$\times$		$\times$		$\times$		$\times$		$\times$	
2_3		$\times$		$\times$		$\times$		$\times$		$\times$		$\times$
2_4	$\times$		$\times$		$\times$		$\times$		$\times$		$\times$	

This arrangement has zero families.

A chiral embedding is $\mathbf{4} = (2_1, 2_2)$ giving rise to the fermions:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_1									$\times$	$\times$		
1_2										$\times$	$\times$	
1_3											$\times$	$\times$
1_4									$\times$			$\times$
1_5									$\times$	$\times$		
1_6										$\times$	$\times$	
1_7											$\times$	$\times$
1_8									$\times$			$\times$
2_1	$\times$	$\times$			$\times$	$\times$			$\times$	$\times$		
2_2		$\times$	$\times$			$\times$	$\times$			$\times$	$\times$	
2_3			$\times$	$\times$			$\times$	$\times$			$\times$	$\times$
2_4	$\times$			$\times$	$\times$			$\times$	$\times$			$\times$

If we identify $SU(2)_L$ as the diagonal subgroup of $SU(2)_{1,2,5,6}$ and $SU(2)_R$ as the diagonal subgroup of $SU(2)_{3,4,7,8}$, then identify the $\mathbf{4}$ of $SU(4)$ with the $\mathbf{4}$ of $SU(4)_{2,3}$ and the $\bar{\mathbf{4}}$ of $SU(4)_{1,4}$, this model has eight chiral families under $SU(2)_L \times SU(2)_R \times SU(4)$.

Group 24/15; also designated $D_4 \tilde{\times} Z_3$

The group 24/15 has nine inequivalent irreducible representations, four singlets and five doublets.

With the embedding $\mathbf{4} = (2_3, 2_5)$, the fermion table is:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1							$\times$		$\times$
1_2								$\times$	$\times$
1_3							$\times$		$\times$
1_4								$\times$	$\times$
2_1							$\times$	$\times\times$	$\times$
2_2							$\times\times$	$\times$	$\times$
2_3		$\times$		$\times$	$\times\times$	$\times$			
2_4	$\times$		$\times$		$\times$	$\times\times$			
2_5	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			

Identifying $SU(2)_L \equiv SU(2)_{1,3}$ and $SU(2)_R \equiv SU(2)_{2,4}$ gives rise to two chiral families.

Another chiral embedding is $\mathbf{4} = (1_2, 1_3, 2_3)$ which gives the chiral fermions:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1		$\times$	$\times$				$\times$		
1_2	$\times$			$\times$				$\times$	
1_3	$\times$			$\times$			$\times$		
1_4		$\times$	$\times$					$\times$	
2_1					$\times$	$\times$		$\times$	$\times$
2_2					$\times$	$\times$	$\times$		$\times$
2_3		$\times$		$\times$	$\times$		$\times$	$\times$	
2_4	$\times$		$\times$			$\times$	$\times$	$\times$	
2_5					$\times$	$\times$			$\times\times$

Identifying $SU(2)_L$ with the diagonal subgroup of 1_1 and 1_3, $SU(2)_R$ with 1_2 and 1_4, and then identifying $2_3 = \mathbf{4}$ and $2_4 = \bar{\mathbf{4}}$ and finally breaking the other three $SU(4)$'s gives rise to six chiral families.

As an alternative 24/15 model we can embed $\mathbf{4} = (2_3, 2_3)$ and obtain:

	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1							$\times\times$		
1_2								$\times\times$	
1_3							$\times\times$		
1_4								$\times\times$	
2_1								$\times\times$	$\times\times$
2_2							$\times\times$		$\times\times$
2_3		$\times\times$		$\times\times$	$\times\times$				
2_4	$\times\times$		$\times\times$			$\times\times$			
2_5					$\times\times$	$\times\times$			

With $SU(2)_L$, $SU(2)_R$ as diagonal subgroups of $SU(2)_1 \times SU(2)_3$ and $SU(2)_2 \times SU(2)_4$ respectively, and breaking completely $SU(4)_4$, this leads to four chiral families.

<u>g = 27.</u>

The non-pseudoreal groups number two and both are SDPGs. In the notation of Thomas and Wood [12] they are: $27/4, 5$. So we now treat these in the order they are enumerated by Thomas and Wood.

<u>Group 27/4; also designated $Z_9 \tilde{\times} Z_3$</u>

$27/4$ has nine singlet $1_1, ..., 1_9$ and two triplet $3_1, 3_2$ irreducible representations.

We may choose the embedding $4 = (1_2, 3_1)$. The chiral fermions are:

	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1		$\times$								$\times$	
1_2			$\times$							$\times$	
1_3	$\times$									$\times$	
1_4					$\times$					$\times$	
1_5						$\times$				$\times$	
1_6				$\times$						$\times$	
1_7								$\times$		$\times$	
1_8									$\times$	$\times$	
1_9							$\times$			$\times$	
3_1										$\times$	$\times \times \times$
3_2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$

Putting $N = 2$, the gauge group is $SU(2)^9 \times SU(6)_1 \times SU(6)_2$ and the chiral fermions are, from the above table:

$$(\sum_{i=1}^{i=9} 2_i, \bar{6}_1) + (6_1, \bar{6}_1 + 3(\bar{6_2})) + (6_2, \sum_{i=1}^{i=9} 2_i) + (6_2, \bar{6_2}) \tag{23}$$

Though asymmetric in representations, this result is anomaly-free with respect both to $SU(6)_1$ and $SU(6)_2$.

Group 27/5; also designated $(Z_3 \times Z_3)\tilde{\times} Z_3$

The multiplication tables, and hence the model-building, are identical for 27/4 and 27/5. The group 27/5 merits no further separate discussion.

g = 30.

The non-pseudoreal groups number two, and neither is an SDPG. In the notation of Thomas and Wood [12], they are: $30/2, 3$. So we now treat these in the order they are enumerated by Thomas and Wood.

Group 30/2; also designated $D_5 \times Z_3$

30/2 has six singlets $1\alpha^i, 1^{'}\alpha^i$ and six doublets $2\alpha^i, 2^{'}\alpha^i$ with $\alpha = exp(i\pi/3)$ and i = 0, 1, 2.

Choosing $\mathbf{4} = (1\alpha, 1^{'}, 2\alpha)$ yields as fermions:

	1	$1^{'}$	2	$2^{'}\alpha$	1α	$1^{'}\alpha$	2α	$2^{'}\alpha$	$1\alpha^2$	$1^{'}\alpha^2$	$2\alpha^2$	$2^{'}\alpha^2$
1		$\times$			$\times$		$\times$					
$1^{'}$	$\times$					$\times$	$\times$					
2			$\times$		$\times$	$\times$	$\times$	$\times$				
$2^{'}$				$\times$			$\times$	$\times\times$				
1α						$\times$			$\times$		$\times$	
$1^{'}\alpha$					$\times$					$\times$	$\times$	
2α							$\times$		$\times$	$\times$	$\times$	$\times$
$2^{'}\alpha$								$\times$			$\times$	$\times\times$
$1\alpha^2$	$\times$		$\times$							$\times$		
$1^{'}\alpha^2$		$\times$	$\times$						$\times$			
$2\alpha^2$	$\times$	$\times$	$\times$	$\times$							$\times$	
$2^{'}\alpha^2$			$\times$	$\times\times$								$\times$

Identify $SU(2)_L$ with the diagonal subgroup of $SU(2)_1 \times SU(2)_2$ (associated with $1, 1^{'}$) and $SU(2)_R$ with the diagonal subgroup of $SU(2)_5 \times SU(2)_6$ (associated with $1\alpha^2, 1^{'}\alpha^2$); break the $SU(4)$s associated with $2, 2\alpha^2$ to arrive at two chiral families.

<u>Group 30/3; also designated $D_3 \times Z_5$</u>

This group has irreps which comprise ten singlets and five doublets and yields, for $N = 2$, the gauge group $SU(2)^{10} \times SU(4)^5$.

As we have encountered for groups $D_3 \times Z_p$ (with $g = 6p$) the embedding $\mathbf{4} = (1\alpha^{a_1}, 1^{'}\alpha^{a_2}, 2\alpha^{a_3})$ must satisfy $a_1 + a_2 = -2a_3$ (mod p) for consistency, as well as $a_1 \neq 0$ to ensure $\mathcal{N} = 0$.

There are several interesting such examples, one of which is $\mathbf{4} = (1\alpha, 1^{'}, 2\alpha^2)$ which gives as fermions:

	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$1\alpha^3$	$1'\alpha^3$	$2\alpha^3$	$1\alpha^4$	$1'\alpha^4$	$2\alpha^4$
1		$\times$		$\times$					$\times$						
$1'$	$\times$				$\times$				$\times$						
2			$\times$			$\times$	$\times$	$\times$	$\times$						
1α					$\times$		$\times$					$\times$			
$1'\alpha$				$\times$				$\times$				$\times$			
2α						$\times$			$\times$	$\times$	$\times$	$\times$			
$1\alpha^2$								$\times$		$\times$					$\times$
$1'\alpha^2$							$\times$				$\times$				$\times$
$2\alpha^2$									$\times$			$\times$	$\times$	$\times$	$\times$
$1\alpha^3$			$\times$								$\times$		$\times$		
$1'\alpha^3$			$\times$							$\times$				$\times$	
$2\alpha^3$	$\times$	$\times$	$\times$									$\times$			$\times$
$1\alpha^4$	$\times$					$\times$								$\times$	
$1'\alpha^4$		$\times$				$\times$							$\times$		
$2\alpha^4$			$\times$	$\times$	$\times$	$\times$									$\times$

In an obvious notation, the chiral fermions are:

$$(2_1 + 2_2, \bar{4}_3 + 4_4) + (2_3 + 2_4, \bar{4}_4 + 4_5) + (2_5 + 2_6, \bar{4}_5 + 4_1) + (2_7 + 2_8, \bar{4}_1 + 4_2) + (2_9 + 2_{10}, \bar{4}_2 + 4_3) \tag{24}$$

By identifying, for example (there are equivalent cyclic permutations) $SU(2)_L$ as the diagonal subgroup of $SU(2)_1 \times SU(2)_2 \times SU(2)_7 \times SU(2)_8$, $SU(2)_R$ as the diagonal subgroup of

$SU(2)_5 \times SU(2)_6 \times SU(2)_9 \times SU(2)_{10}$, generalized color $SU(4)$ as the diagonal subgroup of $SU(4)_1 \times SU(4)_3$, and breaking completely $SU(4)_{2,4,5}$ give rise to four chiral families.

We can examine the infinite series $D_3 \times Z_p$ for $p \geq 3$ (as necessary for non-pseudoreality). The order is $g = 6p$. By generalizing the above discussions of 18/3 ($D_3 \times Z_3$), 24/9 ($D_3 \times Z_4$) and 30/3 ($D_3 \times Z_5$) we find that with the same type of embedding one arrives at a maximal number of 2[p] chiral families where [x] is the largest integer not greater than x. For example, with p = 3, 4, 5, 6, 7, 8, 9, 10,.... one obtains 2, 4, 4, 6, 6, 8, 8, 10.... chiral families resspectively. This is an example of accessing the more difficult nonabelian Γ with $g \geq 32$ at least for orders $g = 6p \geq 36$.

That completes the analysis of the occurrence of chiral fermions for Γ with $g \leq 31$. For the cases where there are ≥ 3 chiral families, it remains to check whether the spectrum of complex scalars is sufficient to allow spontaneous symmetry breaking to the Standard Model gauge group.

This is the subject of the next two sections.

V. THE SCALAR SECTOR

In order to carry out the spontaneous symmetry breaking (SSB) in the chiral models we found in the last section, we must first extract the scalar sector from eq. (5), where the 6 is gotten from the embedding of $(\mathbf{4} \times \mathbf{4})_A$ which in turn follows from the embedding of the $\mathbf{4}$. We only consider models of phenomenological interest, i.e., those which potentially have three or more families, but preferably three. With this perspective in mind we first collect the models, they are:

$\underline{16/8 \text{ with } \mathbf{4} = (\mathbf{2}_1, \mathbf{2}_1)}$ and $\chi = 2^8$ with $N = 2$.

$\underline{16/8 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_5, \mathbf{2}_1)}$ and $\chi = 2^7$ with $N = 2$.

$\underline{16/11 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2)}$ and $\chi = 432$ with $N = 3$.

$\underline{16/11 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_4)}$ and $\chi = 216$ with $N = 3$.

$\underline{16/13 \text{ with } \mathbf{4} = (\mathbf{1}_3, \mathbf{1}_4, \mathbf{2}_1)}$ and $\chi = 2^6$ with $N = 2$.

$\underline{16/13 \text{ with } \mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)}$ and $\chi = 2^6$ with $N = 2$

$\underline{16/13 \text{ with } \mathbf{4} = (\mathbf{2}_1, \mathbf{2}_1)}$ and $\chi = 2^7$ with $N = 2$.

$\underline{18/3 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha)}$ and $\chi = 192$ with $N = 2$.

$\underline{20/5 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2)}$ and $\chi = 144$ with $N = 3$.

$\underline{20/5 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_4)}$ and $\chi = 72$ with $N = 3$.

$\underline{21/2 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)}$ and $\chi = 108$ with $N = 2$.

$\underline{24/7 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha)}$ and $\chi = 240$ with $N = 2$.

$\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{1}_1\alpha, \mathbf{1}_2\alpha^3, \mathbf{2}\alpha^2)}$ and $\chi = 320$ with $N = 2$.

$\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{1}_1\alpha, \mathbf{1}_2\alpha, \mathbf{2}\alpha)}$ and $\chi = 320$ with $N = 2$.

$\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{1}_1\alpha^2, \mathbf{1}_2, \mathbf{2}\alpha)}$ and $\chi = 192$ with $N = 2$.

$\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{2}\alpha, \mathbf{2}\alpha)}$ and $\chi = 384$ with $N = 2$.

$\underline{24/13 \text{ with } \mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)}$ and $\chi = 48$ with $N = 2$.

$\underline{24/14 \text{ with } \mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)}$ and $\chi = 192$ with $N = 2$.

$\underline{24/15 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_3, \mathbf{2}_3)}$ and $\chi = 2^7$ with $N = 2$.

$\underline{24/15 \text{ with } \mathbf{4} = (\mathbf{2}_3, \mathbf{2}_5)}$ and $\chi = 2^7$ with $N = 2$.

$\underline{24/15 \text{ with } \mathbf{4} = (\mathbf{2}_3, \mathbf{2}_3)}$ and $\chi = 2^8$ with $N = 2$.

$\underline{27/4 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)}$ and $\chi = 324$ with $N = 2$.

$\underline{30/2 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha)}$ and $\chi = 336$ with $N = 2$.

$\underline{30/3 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha^2)}$ and $\chi = 320$ with $N = 2$.

First we consider $\underline{16/8 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{2}_1)}$, where we have included this example to demonstrate improper embedding. This representation is complex and would be expected to lead to chiral fermions, but $\mathbf{6} = (\mathbf{4} \times \mathbf{4})_A = \mathbf{1}_1 + 2(\mathbf{2}_1 + \mathbf{2}_1) + (\mathbf{1}_5 + \mathbf{1}_6 + \mathbf{1}_7 + \mathbf{1}_8)_A$ is complex (for any choice of singlet in the last parenthetical expression), and therefore the embedding $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{2}_1)$ is improper and we need not consider this or other such models further.

Let us define the chirality measure χ of a model as the number of chiral fermion states. This variable applies to any irreps and provides a somewhat finer measure of chirality than the number of families. As spontaneous symmetry breaking (SSB) proceeds, χ decreased (except under unusual circumstances). For instance, the standard model and minimal $SU(5)$ both have $\chi = 45$ initially. By the time the symmetry is broken to $SU(3) \times U_{EM}(1)$, $\chi = 3$ since the neutrino's cannot acquire mass due to global $B - L$ symmetry. On the other hand, three family $SO(10)$ and E_6 models start with $\chi = 48$ and $\chi = 81$ respectively but both break to $\chi = 0$.

In model building with AdSCFTs we are faced with a number of choices. if we require the initial model be chiral before SSB, then we need $\chi \geq 45$initially. However, since the scale of SSB M_{AdS} in these models can be relatively low (few 10s of TeV), vector like models are more appealing than usual, and we could allow an initial $\chi = 0$ without resorting to incredibly detailed fine tunings. Our prejudice is to still require a chiral model with $\chi \geq 45$ initially in order to gain some control in model building, but we want to make it clear that, even though we have not displayed them explicitly, the entire class of vectorlike model based on the nonabelian orbifold classification given here would be worthy of detailed study. There are also models (chiral or vectorlike) that break from G_{AdS} to $SU(3) \times U_{EM}(1)$ but without going through $SU(3) \times SU(2) \times U(1)$ directly. As M_{AdS} may be not far above M_Z, there may be models in this class that could be in agreement with current data, but again we restrict most of our discussion to chiral models that break through the standard model. What is encouraging is the fact that orbifold AdS/CFTs provide such a wealth of potentially interesting models.

16/8 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_1)$. Here $\mathbf{6} = \; 3(\mathbf{1}_5) + \mathbf{1}_6 + \mathbf{1}_7 + \mathbf{1}_8$ which is real so the embedding is proper and the scalar sector is:

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1					$\times\times\times$	$\times$	$\times$	$\times$		
1_2					$\times$	$\times$	$\times$	$\times$		
1_3					$\times$	$\times$	$\times$	$\times$		
1_4					$\times$	$\times$	$\times$	$\times\times\times$		
1_5	$\times\times\times$	$\times$	$\times$	$\times$						
1_6	$\times$	$\times$	$\times$	$\times$						
1_7	$\times$	$\times$	$\times$	$\times$						
1_8	$\times$	$\times$	$\times$	$\times\times\times$						
2										$\times\times\times$ $\times\times\times$
$2'$									$\times\times\times$ $\times\times\times$	

16/8 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_{4+i}, \mathbf{2}_1)$ and $\mathbf{6} = (\mathbf{1}_{x(i)}, \mathbf{2}, \mathbf{2}', (\mathbf{1_5} + \mathbf{1_6} + \mathbf{1_7} + \mathbf{1_8})_\mathbf{A})$ where $x = 6, 5, 8,$ or 7 for $i = 1, 2, 3, 4$. The fermionic sectors of these models are identical up to permutation, but there are two potential types of scalar sectors, depending on whether $\mathbf{1}_{x(i)}$ is the same as or different from the antisymmetric product $(\mathbf{2}_1 \times \mathbf{2}_1)_A$. Let us relabel the singlets so $(\mathbf{2}_1 \times \mathbf{2}_1)_A = \mathbf{1_6}$, and then choose $\mathbf{1}_{x(i)}$ to be either $\mathbf{1_5}$ or $\mathbf{1_6}$. Now the two inequivalent scalar sectors (In this instance, it is easier to analyse both models and show that neither phenomenology is interesting, rather than untangle the correct antisymmetric singlet in $(\mathbf{2}_1 \times \mathbf{2}_1)_A$. See the next section.) are:

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1					$\times(5)$	(6)			$\times$	$\times$
1_2					(6)	$\times(5)$			$\times$	$\times$
1_3							$\times(5)$	(6)	$\times$	$\times$
1_4							(6)	$\times(5)$	$\times$	$\times$
1_5	$\times(5)$	(6)							$\times$	$\times$
1_6	(6)	$\times(5)$							$\times$	$\times$
1_7			$\times(5)$	(6)					$\times$	$\times$
1_8			(6)	$\times(5)$					$\times$	$\times$
2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times\times$
$2'$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times\times$	

where(5) is replaced by an "$\times$" and (6) by a blank if $\mathbf{1}_{x(i)} = \mathbf{1_5}$ and vis versa if $\mathbf{1}_{x(i)} = \mathbf{1_6}$.

16/11 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2)$ and $\mathbf{6} = (\mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3\ )$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1			$(\times)^6$							
1_2		$(\times)^6$								
1_3	$(\times)^6$									
1_4				$(\times)^6$						
1_5							$(\times)^6$			
1_6						$(\times)^6$				
1_7					$(\times)^6$					
1_8								$(\times)^6$		
2									$(\times)^6$	
$2'$										$(\times)^6$

$\underline{16/11 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_4)}$ and $\mathbf{6} = (\mathbf{1}_1, \mathbf{1}_1, \mathbf{1}_1, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3\ )$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	$(\times)^3$		$(\times)^3$							
1_2		$(\times)^6$								
1_3	$(\times)^3$		$(\times)^3$							
1_4				$(\times)^6$						
1_5					$(\times)^3$		$(\times)^3$			
1_6						$(\times)^6$				
1_7					$(\times)^3$		$(\times)^3$			
1_8								$(\times)^6$		
2									$(\times)^6$	
$2'$										$(\times)^6$

$\underline{16/13 \text{ with } \mathbf{4} = (\mathbf{1_3}, \mathbf{1_4}, \mathbf{2}_1)}$ and $\mathbf{6} = (\mathbf{1}_2, \mathbf{1}_c, \mathbf{2}_1, \mathbf{2}_3)$, where $\mathbf{1}_c = (\mathbf{2}_1 \times \mathbf{2}_1)_{\mathbf{A}}$ so we have $\mathbf{1}_c$ is either $\mathbf{1}_2$ or $\mathbf{1}_3$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1		$\times(2)$	(3)		$\times$		$\times$
1_2	$\times(2)$			(3)	$\times$		$\times$
1_3	(3)			$\times(2)$	$\times$		$\times$
1_4		(3)	$\times(2)$		$\times$		$\times$
2_1	$\times$	$\times$	$\times$	$\times$		$\times\times$	$\times\times$
2_2					$\times\times$	$\times\times$	$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$	$\times\times$	$\times\times$	

<u>16/13 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)$</u> and $\mathbf{6} = (\mathbf{1}_a, \mathbf{1}_b, \mathbf{2}_1, \mathbf{2}_3)$, where $\mathbf{1}_a = (\mathbf{2}_1 \times \mathbf{2}_1)_{\mathbf{A}} = (1_2 + 1_3 + 2_2)_{\mathbf{A}}$ and $\mathbf{1}_b = (\mathbf{2}_2 \times \mathbf{2}_2)_{\mathbf{A}} + (1_1 + 1_2 + 1_3 + 1_4)_{\mathbf{A}}$.

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	(1)	(2)	(3)	(4)	$\times$		$\times$
1_2	(2)	(1)	(4)	(3)	$\times$		$\times$
1_3	(3)	(4)	(1)	(2)	$\times$		$\times$
1_4	(4)	(3)	(2)	(1)	$\times$		$\times$
2_1	$\times$	$\times$	$\times$	$\times$	(1)(4)	$\times\times$	(2)(3)
2_2					$\times\times$	(1)(2) (3)(4)	$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$	(2)(3)	$\times\times$	(1)(4)

where we insert $\times$s at the locations in parenthesis when the singlets are chosen properly from the antisymmetric products of the doublets. There are three inequivalent choices, either (i) put $\times\times$ at location (2), or (ii) put an $\times$ at (2) and one at (3), or (iii) put $\times$ at (2) and $\times$ at (1). All other choices lead to equivalent models. Thus, without detailed knowledge of the antisymmetric products, we can still reduce the analysis to the consideration of these three cases.

16/13 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_1)$ and $\mathbf{6} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_3, \mathbf{2}_2)$ (which is equivalent to $\mathbf{6} = (\mathbf{1}_2, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{2}_2)$ for SSB up to a relabeling of irreps).

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1		$\times\times\times$	$\times$			$\times$	
1_2	$\times\times\times$					$\times$	
1_3	$\times$			$\times\times\times$		$\times$	
1_4			$\times\times\times$			$\times$	
2_1					$\times$		$\times\times$ $\times\times\times$
2_2	$\times$	$\times$	$\times$	$\times$		$\times\times$ $\times\times$	
2_3					$\times\times$ $\times\times\times$		$\times$

18/3 with $4 = (1'\alpha, 1', 2\alpha)$ and $6 = (1'\alpha, 2\alpha, 2\alpha^2, 1'\alpha^2)$

$\otimes$	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$
1					$\times$	$\times$		$\times$	$\times$
$1'$				$\times$		$\times$	$\times$		$\times$
2				$\times$	$\times$	$\times\times$	$\times$	$\times$	$\times\times$
1α		$\times$	$\times$					$\times$	$\times$
$1'\alpha$	$\times$		$\times$				$\times$		$\times$
2α	$\times$	$\times$	$\times\times$				$\times$	$\times$	$\times\times$
$1\alpha^2$		$\times$	$\times$		$\times$	$\times$			
$1'\alpha^2$	$\times$		$\times$	$\times$		$\times$			
$2\alpha^2$	$\times$	$\times$	$\times\times$	$\times$	$\times$	$\times\times$			

20/5 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2)$ and $\mathbf{6} = (\mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3\ )$ is very much like the 16/11 model with the similar embedding. Here

$\otimes$	1_1	1_2	1_3	1_4	4
1_1			$(\times)^6$		
1_2				$(\times)^6$	
1_3	$(\times)^6$				
1_4		$(\times)^6$			
4					

and a VEV for any of these renders the entire fermion sector vectorlike.

For 20/5 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_2, \mathbf{1}_4)$ and $\mathbf{6} = (\mathbf{1}_1, \mathbf{1}_1, \mathbf{1}_1, \mathbf{1}_3, \mathbf{1}_3, \mathbf{1}_3\ )$ we have the scalars:

$\otimes$	1_1	1_2	1_3	1_4	4
1_1	$\times\times\times$		$\times\times\times$		
1_2		$\times\times\times$		$\times\times\times$	
1_3	$\times\times\times$		$\times\times\times$		
1_4		$\times\times\times$		$\times\times\times$	
4					$(\times)^6$

$\underline{21/2 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)}$ and $\mathbf{6} = \mathbf{3}_1 + \mathbf{3}_2$ with $N = 2$. (All other embeddings of the $\mathbf{4}$ with chiral fermions and $\aleph = 0$ SUSY are permutations and therefore equivalent to this model.). Here $\mathbf{6}$ is real so the embedding is proper and the scalar sector is:

$\otimes$	1_1	1_2	1_3	3_1	3_2
1_1				$\times$	$\times$
1_2				$\times$	$\times$
1_3				$\times$	$\times$
3_1	$\times$	$\times$	$\times$	$\times\times$	$\times\times\times$
3_2	$\times$	$\times$	$\times$	$\times\times\times$	$\times\times$

24/7 or equivalently 24/8 (since they have isomorphic irrep product tables) with $\mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1}_2, \mathbf{2}\alpha)$ and $\mathbf{6} = (\mathbf{1}_2\alpha, \mathbf{1}_2\alpha^2, \mathbf{2}\alpha, \mathbf{2}\alpha^2)$

$\otimes$	1_1	1_2	1_3	1_4	2	$1_1\alpha$	$1_2\alpha$	$1_3\alpha$	$1_4\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$1_3\alpha^2$	$1_4\alpha^2$	$2\alpha^2$
1_1							$\times$			$\times$		$\times$			$\times$
1_2						$\times$				$\times$	$\times$				$\times$
1_3									$\times$	$\times$				$\times$	$\times$
1_4								$\times$		$\times$			$\times$		$\times$
2						$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$1_1\alpha$		$\times$			$\times$							$\times$			$\times$
$1_2\alpha$	$\times$				$\times$						$\times$				$\times$
$1_3\alpha$				$\times$	$\times$									$\times$	$\times$
$1_4\alpha$			$\times$		$\times$								$\times$		$\times$
2α	$\times$	$\times$	$\times$	$\times$	$\times$						$\times$	$\times$	$\times$	$\times$	$\times$
$1_1\alpha^2$		$\times$			$\times$		$\times$			$\times$					
$1_2\alpha^2$	$\times$				$\times$	$\times$				$\times$					
$1_3\alpha^2$				$\times$	$\times$				$\times$	$\times$					
$1_4\alpha^2$			$\times$		$\times$			$\times$		$\times$					
$2\alpha^2$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$					

The scalars for $\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1_2}\alpha^{\mathbf{3}}, \mathbf{2}\alpha^{\mathbf{2}})}$ and $\mathbf{6} = (\mathbf{1}_2, \mathbf{1}_2, \mathbf{2}\alpha, \mathbf{2}\alpha^3)$ are:

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1		$\times\times$				$\times$						$\times$
1_2	$\times\times$					$\times$						$\times$
2			$\times\times$	$\times$	$\times$	$\times$				$\times$	$\times$	$\times$
$1_1\alpha$			$\times$		$\times\times$				$\times$			
$1_2\alpha$			$\times$	$\times\times$					$\times$			
2α	$\times$	$\times$	$\times$			$\times\times$	$\times$	$\times$	$\times$			
$1_1\alpha^2$						$\times$		$\times\times$				$\times$
$1_2\alpha^2$						$\times$	$\times\times$					$\times$
$2\alpha^2$				$\times$	$\times$	$\times$			$\times\times$	$\times$	$\times$	$\times$
$1_1\alpha^3$			$\times$						$\times$		$\times\times$	
$1_1\alpha^3$			$\times$						$\times$	$\times\times$		
$2\alpha^3$	$\times$	$\times$	$\times$				$\times$	$\times$	$\times$			$\times\times$

The scalars for $\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1_2}\alpha, \mathbf{2}\alpha)}$ and $\mathbf{6} = (\mathbf{1_2}\alpha, \mathbf{2}, \mathbf{1_2}\alpha, \mathbf{2}\alpha^{\mathbf{2}})$ are:

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1								$\times\times$	$\times\times$			
1_2							$\times\times$		$\times\times$			
2							$\times\times$	$\times\times$	$\times\times$ $\times\times$			
$1_1\alpha$											$\times\times$	$\times\times$
$1_2\alpha$										$\times\times$		$\times\times$
2α										$\times\times$	$\times\times$	$\times\times$ $\times\times$
$1_1\alpha^2$		$\times\times$	$\times\times$									
$1_2\alpha^2$	$\times\times$		$\times\times$									
$2\alpha^2$	$\times\times$	$\times\times$	$\times\times$ $\times\times$									
$1_1\alpha^3$					$\times\times$	$\times\times$						
$1_1\alpha^3$				$\times\times$		$\times\times$						
$2\alpha^3$				$\times\times$	$\times\times$	$\times\times$ $\times\times$						

For 24/9 with $\mathbf{4} = (\mathbf{1}_1\alpha, \mathbf{1}_2, \mathbf{2}\alpha)$ and $\mathbf{6} = (\mathbf{1}_2\alpha^2, \mathbf{2}\alpha, \mathbf{2}\alpha^{-1}, \mathbf{1}_2\alpha^{-2})$ where $\alpha^4 = 1$, the scalar sector is:

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1						$\times$		$\times\times$				$\times$
1_2						$\times$	$\times\times$					$\times$
2				$\times$	$\times$	$\times$			$\times\times$	$\times$	$\times$	$\times$
$1_1\alpha$			$\times$						$\times$		$\times\times$	
$1_2\alpha$			$\times$						$\times$	$\times\times$		
2α	$\times$	$\times$	$\times$				$\times$	$\times$	$\times$			$\times\times$
$1_1\alpha^2$		$\times\times$				$\times$						$\times$
$1_2\alpha^2$	$\times\times$					$\times$						$\times$
$2\alpha^2$			$\times\times$	$\times$	$\times$	$\times$				$\times$	$\times$	$\times$
$1_1\alpha^3$			$\times$		$\times\times$				$\times$			
$1_2\alpha^3$			$\times$	$\times\times$					$\times$			
$2\alpha^3$	$\times$	$\times$	$\times$			$\times\times$	$\times$	$\times$	$\times$			

For $\underline{24/9 \text{ with } \mathbf{4} = (\mathbf{2}\alpha, \mathbf{2}\alpha)}$ where $\mathbf{6} = 3(\mathbf{1}_2\alpha^2) + \mathbf{1}_1\alpha^2 + \mathbf{2}\alpha^2$,the scalars are:

$\otimes$	1_1	1_2	2	$1_1\alpha$	$1_2\alpha$	2α	$1_1\alpha^2$	$1_2\alpha^2$	$2\alpha^2$	$1_1\alpha^3$	$1_2\alpha^3$	$2\alpha^3$
1_1							$\times$	$\times\times\times$	$\times\times$			
1_2							$\times\times\times$	$\times$	$\times\times$			
2							$\times\times$	$\times\times$	$\times\times\times$ $\times\times\times$			
$1_1\alpha$										$\times$	$\times\times\times$	$\times\times$
$1_2\alpha$										$\times\times\times$	$\times$	$\times\times$
2α										$\times\times$	$\times\times$	$\times\times\times$ $\times\times\times$
$1_1\alpha^2$	$\times$	$\times\times\times$	$\times\times$									
$1_2\alpha^2$	$\times\times\times$	$\times$	$\times\times$									
$2\alpha^2$	$\times\times$	$\times\times$	$\times\times\times$ $\times\times\times$									
$1_1\alpha^3$				$\times$	$\times\times\times$	$\times\times$						
$1_2\alpha^3$				$\times\times\times$	$\times$	$\times\times$						
$2\alpha^3$				$\times\times$	$\times\times$	$\times\times\times$ $\times\times\times$						

The next example of interest is 24/13 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)$ and $\mathbf{6} = \mathbf{1}_1 + \mathbf{1}_2 + \mathbf{1}_3 + \mathbf{3}$ with scalars:

$\otimes$	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1	$\times$	$\times$	$\times$				$\times$
1_2	$\times$	$\times$	$\times$				$\times$
1_3	$\times$	$\times$	$\times$				$\times$
2_1				$\times\times$	$\times\times$	$\times\times$	
2_2				$\times\times$	$\times\times$	$\times\times$	
2_3				$\times\times$	$\times\times$	$\times\times$	
3	$\times$	$\times$	$\times$				$\times\times$

There are two ineqivalent models for the group 24/15, they are $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_3, \mathbf{2}_3)$ where $\mathbf{6} = \mathbf{1}_4 + \mathbf{1}_{2[4]} + \mathbf{2}_3 + \mathbf{2}_4$ and the scalars are:

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1				$\times\times$			$\times$	$\times$	
1_2			$\times\times$				$\times$	$\times$	
1_3		$\times\times$					$\times$	$\times$	
1_4	$\times\times$						$\times$	$\times$	
2_1						$\times\times$	$\times$	$\times$	$\times\times$
2_2					$\times\times$		$\times$	$\times$	$\times\times$
2_3	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times\times$	
2_4	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times\times$		
2_5					$\times\times$	$\times\times$			$\times\times$

if $(\mathbf{2}_3 \times \mathbf{2}_3)_A = \mathbf{1}_4$ but if it is $\mathbf{1}_2$ then the top 4×4 changes to:

	$\times$		$\times$
$\times$		$\times$	
	$\times$		$\times$
$\times$		$\times$	

The other $\underline{24/15}$ case has $\underline{\mathbf{4} = (\mathbf{2}_3, \mathbf{2}_3)}$ where $\mathbf{6} = 3(\mathbf{1}_2) + \mathbf{1}_4 + \mathbf{2}_1$ and the scalars are (this time swapping $\mathbf{1}_2$ and $\mathbf{1}_4$ gives equivalent models):

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1		$\times\times\times$		$\times$	$\times$				
1_2	$\times\times\times$		$\times$			$\times$			
1_3		$\times$		$\times\times\times$	$\times$				
1_4	$\times$		$\times\times\times$						
2_1	$\times$		$\times$		$\times$	$\times\times$ $\times\times$			
2_2		$\times$			$\times\times$ $\times\times$	$\times$			
2_3								$\times\times\times$ $\times\times$	$\times$
2_4							$\times\times\times$ $\times\times$		$\times$
2_5							$\times$	$\times$	$\times\times$ $\times\times$

The next model to evaluate is $\underline{27/4 \text{ with } \mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)}$, where $\mathbf{6} = \mathbf{3}_1 + \mathbf{3}_2$ is real. The scalar sector is:

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1										$\times$	$\times$
1_2										$\times$	$\times$
1_3										$\times$	$\times$
1_4										$\times$	$\times$
1_5										$\times$	$\times$
1_6										$\times$	$\times$
1_7										$\times$	$\times$
1_8										$\times$	$\times$
1_9										$\times$	$\times$
3_1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times\times\times$
3_2	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times\times\times$	

And finally at order 30 we have for $\underline{30/2 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha)}$ and $\mathbf{6} = (\mathbf{1}'\alpha + \mathbf{2}\alpha + \mathbf{2}\alpha^{-1} + \mathbf{1}'\alpha^{-1})$ where $\alpha^3 = 1$, a model with scalar sector:

$\otimes$	1	$1'$	2	$2'$	1α	$1'\alpha$	2α	$2'\alpha$	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$2'\alpha^2$
1						$\times$	$\times$			$\times$	$\times$	
$1'$					$\times$		$\times$		$\times$		$\times$	
2					$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$2'$							$\times$	$\times\times$			$\times$	$\times\times$
1α		$\times$	$\times$							$\times$	$\times$	
$1'\alpha$	$\times$		$\times$						$\times$		$\times$	
2α	$\times$	$\times$	$\times$	$\times$					$\times$	$\times$	$\times$	$\times$
$2'\alpha$			$\times$	$\times\times$							$\times$	$\times\times$
$1\alpha^2$		$\times$	$\times$			$\times$	$\times$					
$1'\alpha^2$	$\times$		$\times$		$\times$		$\times$					
$2\alpha^2$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$				
$2'\alpha^2$			$\times$	$\times\times$			$\times$	$\times\times$				

The other possibility at order 30 is $\underline{30/3 \text{ with } \mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha^2)}$ where $\mathbf{6} = \mathbf{1}'\alpha + \mathbf{2}\alpha^2 + \mathbf{2}\alpha^3 + \mathbf{1}'\alpha^4$ and $\alpha^5 = 1$, where the scalars are:

$\otimes$	1	$1'$	2	1α	$1'\alpha$	2α	$1\alpha^2$	$1'\alpha^2$	$2\alpha^2$	$1\alpha^3$	$1'\alpha^3$	$2\alpha^3$	$1\alpha^4$	$1'\alpha^4$	$2\alpha^4$
1					$\times$				$\times$			$\times$		$\times$	
$1'$				$\times$					$\times$			$\times$	$\times$		
2						$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$
1α		$\times$						$\times$				$\times$			$\times$
$1'\alpha$	$\times$						$\times$					$\times$			$\times$
2α			$\times$						$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$1\alpha^2$			$\times$		$\times$						$\times$				$\times$
$1'\alpha^2$			$\times$	$\times$						$\times$					$\times$
$2\alpha^2$	$\times$	$\times$	$\times$			$\times$						$\times$	$\times$	$\times$	$\times$
$1\alpha^3$			$\times$			$\times$		$\times$						$\times$	
$1'\alpha^3$			$\times$			$\times$	$\times$						$\times$		
$2\alpha^3$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$						$\times$
$1\alpha^4$		$\times$				$\times$			$\times$		$\times$				
$1'\alpha^4$	$\times$					$\times$			$\times$	$\times$					
$2\alpha^4$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$			

VI. SPONTANEOUS SYMMETRY BREAKING

We are now in a position to carry out the spontaneous symmetry breaking for the models with fermions and scalars given in the previous two sections. We restrict ourselves to chiral models with the potential of at least three families ($\chi \geq 45$) and for the most part consider only models with $N = 2$, although we have included two $N = 3$ models. Again, we move progressively through the models of increasing order of Γ. The model is completely fixed by Γ, the embedding of $\mathbf{4}$ in Γ, and the choice of N. the first relevant model is:

<u>16/8 with $\mathbf{4} = (\mathbf{2_1},\mathbf{2_1})$ and $N = 2$</u>

The chiral fermions are

$2[(2,1,1,1,1,1,1,1;4,1) + (1,1,1,1,2,1,1,1;1,4) + (1,2,1,1,1,1,1,1;4,1) + (1,1,1,1,1,2,1,1;1,4)$
$+ (1,1,2,1,1,1,1,1;4,1) + (1,1,1,1,1,1,2,1;1,4) + (1,1,1,2,1,1,1,1;4,1) + (1,1,1,1,1,1,1,2;1,4)$
$+ (2,1,1,1,1,1,1,1;1,\bar{4}) + (1,1,1,1,2,1,1,1;\bar{4},1) + (1,2,1,1,1,1,1,1;1,\bar{4}) + (1,1,1,1,1,2,1,1;\bar{4},1)$
$+ (1,1,2,1,1,1,1,1;1,\bar{4}) + (1,1,1,1,1,1,2,1;\bar{4},1) + (1,1,1,2,1,1,1,1;1,\bar{4}) + (1,1,1,1,1,1,1,2;\bar{4},1)]$ and $\chi = 2^8$. From the table of scalars for this model, we find that if we break $SU(4) \times SU(4)$ to the diagonal $SU_D(4)$, then the model becomes vectorlike.

All scalars that are nontrivial in the $SU(4)$s are of the form $(1,1,1,1,1,1,1,1;4,\bar{4})+h.c.$, and a VEV for any one can be rotated such that the unbroken symmetry is $SU_D(4)$. All other scalars are $SU_i(2) \times SU_j(2)$ bilinears, hence we cannot break to a Pati-Salam (PS) model or any standard type chiral model.

<u>16/8 with $\mathbf{4} = (\mathbf{1_2}, \mathbf{1_{4+i}}, \mathbf{2_1})$ and $N = 2$,</u>, where $\mathbf{6} = (\mathbf{1_{x(i)}}, \mathbf{2_1}, \mathbf{2_2}, (\mathbf{1_5}, \mathbf{1_6}, \mathbf{1_7}, \mathbf{1}_8)_{\mathbf{A}})$ with $x = 6, 5, 8, 7$ for $i = 1, 2, 3, 4$.

These models have only half the initial chirality of the previous model ($\chi = 2^7$), and the fermions are given above if the overall factor of 2 is removed. As above, we need to break one $SU(4)$, either will do. We choose $SU_2(4)$. For the scalars shown, we can do this with, say, $(1, 1, 1, 2, 1, 1, 1, 1; 1, \bar{4})$ and $(1, 1, 1, 1, 1, 1, 1, 2; 1, 4)$ VEVs. The remaining chiral fermion sector is

$(2, 1, 1, 1, 1, 1; 4) + (1, 1, 1, 2, 1, 1; \bar{4}) + (1, 2, 1, 1, 1, 1; 4)$

$+ (1, 1, 1, 1, 2, 1; \bar{4}) + (1, 1, 2, 1, 1, 1; 4) + (1, 1, 1, 1, 1, 2; \bar{4})$

for $G = \prod_k SU_k(2) \times SU(4)$, with k=1,2,3,5,6,7.

There are only $SU_i(2) \times SU_j(2)$ bilinear scalars of the form$(2_i, 2_j)$ where $i = 1, 2$, or 3 and $j = 4, 5$,or 6, who's VEVs reduce chirality further, so we cannot reach a three-family P-S model.

Note: what one would need is bilinears that allow us to break $SU_1(2) \times SU_2(2) \times SU_3(2)$ to a diagonal subgroup $SU_L(2)$, and similarly for $SU_4(2) \times SU_5(2) \times SU_6(2)$ to $SU_R(2)$. This would then have been a three-family P-S model.

<u>16/11 with $\mathbf{4} = (\mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2})$ and $N = 3$</u>

This model is highly chiral, with $\chi = 432$, and the chiral fermions are $6[(3, \bar{3}, 1, 1, 1, 1, 1, 1; 1, 1) + (1, 1, 1, 1, 3, \bar{3}, 1, 1; 1, 1) + (1, 3, \bar{3}, 1, 1, 1, 1, 1; 1, 1) + (1, 1, 1, 1, 1, 3, \bar{3}, 1; 1, 1)$
$+ (1, 1, 3, \bar{3}, 1, 1, 1, 1; 1, 1) +$
$(1, 1, 1, 1, 1, 1, 3, \bar{3}; 1, 1) + (\bar{3}, 1, 1, 3, 1, 1, 1, 1; 1, 1) + (1, 1, 1, 1, \bar{3}, 1, 1, 3; 1, 1)]$. We can ignore the $SU(6) \times SU(6)$ sector, since it can be broken completely without affecting the chirality. If we

then give VEVs to $(1,1,1,8,1,1,1,1)$ and $(1,1,1,1,1,1,1,8)$ representations of $SU(3)^8$, we arrive at $6[(3,\bar{3},1)+(1,3,\bar{3})+(1,1,3)+(\bar{3},1,1)]$ in the $SU_{i+1}(3)\times SU_{i+2}(3)\times SU_{i+3}(3)$ sector for both $i=0$ and $i=1$. The $i=0$ sector can be broken completely with $(1,1,1,1,8,1)$-type VEVs plus $(1,1,1,3,1,\bar{3})$-type VEVs. The remaining fermions falling nearly into six $E_6 \longrightarrow SU(3)\times SU(3)\times SU(3)$-type families. While close, this model is still unsuccessful.

<u>16/11 with $\mathbf{4}=(\mathbf{1_2},\mathbf{1_2},\mathbf{1_2},\mathbf{1_4})$ and $N=3$</u>

The chiral fermion sector is exactly half the previous case. Again we break $SU(6)\times SU(6)$ completely. Then breaking $\prod_{j=4}^{8} SU_j(3)$ completely with $SU_j(3)$ octet VEVs gives us finally a chiral fermion sector $3[(3,\bar{3},1)+(1,3,\bar{3})+(1,1,3)+(\bar{3},1,1)]$. This is tantalizingly close to the three-family model we seek, but still no cigar!

16/13: There are three potential models for this group.

Consider first the case with

<u>$\mathbf{4}=(2_1,2_1)$ and $N=2$.</u>

Here $\mathbf{6}=(1_2,1_2,1_2,1_3,2_2)$ and the chiral fermions are

$$2[(2,1,1,1;4,1,1) + (1,2,1,1;1,1,4) + (1,1,2,1;1,1,4) + (1,1,1,2;4,1,1) + (2,1,1,1;1,1,\bar{4}) + (1,2,1,1;\bar{4},1,1) + (1,1,2,1;\bar{4},1,1) + (1,1,1,2;1,1,\bar{4})]$$

VEVs of the form $<4_2,\bar{4}_2>$ etc., can break $SU_2(4)$ completely (this group is irrelevant, since there are no chiral fermions with $SU_2(4)$ quantum numbers). VEVs for $(4_1,\bar{4}_3)$ scalars then breaks $SU_1(4)\times SU_3(4)$ to $SU_D(4)$, such that the fermions become vectorlike. On the other hand, VEVs for $(2_4,4_2)+h.c.$ reduces the chiral sector to

$2[(2,1,1;1,4)+(1,2,1;4,1)+(1,1,2;4,1)+2(1,1,1;1,4)$

$+(2,1,1;\bar{4},1)+2(1,1,1;\bar{4},1)+(1,2,1;1,\bar{4})+(1,1,2;1,\bar{4})]$

and then a VEV for (2_3,4_2) + $h.c$ reduces this farther to $2[(2,1;1,4)+(1,2;4,1)+(1,2;1,\bar{4})+(2,1;\bar{4},1)]$.

As above a VEV for $(4_1,\bar{4}_3)$ scalars would render the model vectorlike, while just breaking $SU_3(4)$ would give a one-family model. However, this needs VEVs for (2_1,2_4) and (2_2,2_3), but no scalars of this type exist in the model. We conclude the model has no Pati-Salam type phenomology.

Consider next

<u>**16/13** with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)$ and $N = 2$.</u>

This time **6** is as given in Section 5, but undetermined up to the identification of anti-symmetric singlets in $(\mathbf{2}_i \times \mathbf{2}_i)_A$ with $i = 1,2$. The chiral fermions are as in the $\mathbf{4} = (\mathbf{2_1}, \mathbf{2_1})$ case, but with the overall factor of 2 deleted. A useful strategy is to do a generic spontaneous symmetry breaking analysis to try to obtain a realistic Pati-Salam type phenomenology and then, if successful, one asks if the scalars to carry out the breaking are in the model. As above, $SU_2(4)$ is irrelevant and so can be ignored. If we identify $SU_1(2) \times SU_4(2)$ with $SU_L(2)$ and $SU_2(2) \times SU_3(2)$ with $SU_R(2)$, we find $2[(2,1;1,4)+(1,2;4,1)+(1,2;1,\bar{4})+(2,1;\bar{4},1)]$. Now breaking one of the remaining $SU(4)s$ completely gives two families, and this is the best one can do. Hence independent of what scalars are available, there is no chance to get a model with three or more families.

The remaining 16/13 case is:

<u>$\mathbf{4} = (\mathbf{1_3}, \mathbf{1_4}, \mathbf{2_1})$ with $N = 2$.</u>

Now $\mathbf{6} = (\mathbf{1_2}, \mathbf{2_1}, \mathbf{2_3}, \mathbf{1_c})$. but the chiral fermions are in the same representations as the previous model, and so we can immediately conclude on general grounds that there is no viable phenomenology for this case.

18/3

Now consider

18/3 with $\mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}';, \mathbf{2}\alpha)$ and $N = 3$. This model has $\chi = 192$ and chiral fermions
$(2,1,1,1,1,1;1,4,1) + (1,2,1,1,1,1;1,4,1) + (1,1,2,1,1,1;\bar{4},1,1)$
$+ (1,1,1,2,1,1;\bar{4},1,1) + (1,1,2,1,1,1;1,1,4) + (1,1,1,2,1,1;1,1,4)$
$+ (1,1,1,1,2,1;1,\bar{4},1) + (1,1,1,1,1,2;1,\bar{4},1) + (1,1,1,1,2,1;4,1,1)$
$+ (1,1,1,1,1,2;4,1,1) + (2,1,1,1,1,1;1,1,\bar{4}) + (1,2,1,1,1,1;1,1,\bar{4})$
$+ 2[(1,1,1,1,1,1;\bar{4},4,1) + (1,1,1,1,1,1;1,\bar{4},4) + (1,1,1,1,1,1;4,1,\bar{4})]$. Breaking $SU^6(2)$ to a single diagonal $SU(2)$with all six $(2_i$,$2_j)$ type VEVs of $SU_i(2) \times SU_j(2)$, and then further VEVs of the type (2;4,1,1), (2;1,4,1), and (2;1,1,4) to break the $SU(4)$s to $SU(3)$s leads to the set of remaining chiral fermions:

$2[(3,\bar{3},1) + (1,3,\bar{3}) + (\bar{3},1,3)]$.

So this route leads to two families.

If instead we seek a Pati-Salam model, there are several spontaneous symmetry breaking routes we need to investigate. If we break with (1,1,1,1,1,1;$\bar{4}$, 4, 1) scalars to $SU^6(2) \times SU_D(4) \times SU_3(4)$ we find the fermions remaining chiral are

$(2,1,1,1,1,1;4,1) + (1,2,1,1,1,1;4,1) + (1,1,2,1,1,1;1,4) + (1,1,1,2,1,1;1,4)$
$+ (1,1,2,1,1,1;\bar{4},1) + (1,1,1,2,1,1;\bar{4},1) + (2,1,1,1,1,1;1,\bar{4}) + (1,2,1,1,1,1;1,\bar{4})$.

Now breaking with a $(4_1,\bar{4}_3)$ or $(4_2,\bar{4}_3)$ VEV would render the model vectorlike, so we avoid this and insted give VEVs to $(2_5$,$4_1)$ and $(2_6$,$4_1)$ to break $SU_D(4)$ to $SU'(2)$. However, this yields at most two families.

We must try another route. If we avoid $(\bar{4}, 4)$ type VEVs and give VEVs only to $(2, 4)$ type scalars, we can proceed as follows: $<2_1,4_2>$, $<2_2,4_2>$, $<2_3,\bar{4}_1>$ and $<2_4,\bar{4}_1>$ VEVsbreak $SU^6(2) \times SU^3(4)$ to $SU_5(2) \times SU_6(2) \times SU'(2) \times SU''(2) \times SU(4)$. Some fermions remain chiral but they are insufficient to construct families. We conclude that this model will not provide viable phenomenology.

<u>20/5 with $\mathbf{4} = (\mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2})$ and $N = 3$</u>

The chiral $SU^4(3)$ fermions are $4[(3, \bar{3}, 1, 1)+ (1, 3, \bar{3}, 1) + (1, 1, 3, \bar{3}) + (\bar{3}, 1, 1, 3)]$. (The $SU(6)$ does not participate; it will be ignored.) The only scalars are in representations $(3, 1, \bar{3}, 1)$ +h.c. and $(1, 3, 1, \bar{3})$+h.c. A VEV to, say, the first of these, would break $SU_1(3) \times SU_3(3)$ to a diagonal $SU_D(3)$, and the fermions would become $4[(3, \bar{3}, 1)+ (\bar{3}, 3, 1)+(3, 1, \bar{3})+ (\bar{3}, 1, 3)]$ under $SU_D(3) \times SU_2(3) \times SU_4(3)$, which is vectorlike. Hence any allowed VEVs immediately renders the model vectorlike.

We get no farther with $\mathbf{4} = (\mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2}, \mathbf{1_2})$ and N = 3, where $\mathbf{6} = (\mathbf{1_3}, \mathbf{1_3}, \mathbf{1_3}, \mathbf{1_1}, \mathbf{1_1}, \mathbf{1_1})$, since this model has only half the chirality content of the previous case, and again VEVs will render it vectorlike.

<u>21/2 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)$ and $N = 2$.</u> Now$\mathbf{6} = (\mathbf{3}_1, \mathbf{3}_2)$. (Other embeddings of the $\mathbf{4}$ with n=0 SUSY are permutation of the representations of this model and therefore all equivalent.) The fermions have $\chi = 108$ and are $(2, 1, 1; 6, 1) + (1, 2, 1; 6, 1) + (1, 1, 2; 6, 1) + (2, 1, 1; 1, \bar{6}) + (1, 2, 1; 1, \bar{6}) + (1, 1, 2; 1, \bar{6}) + (1, 1, 1; \bar{6}, 6)$. A VEV for a $(\bar{6}, 6)$ scalar renders the model vectorlike. Our only other option is to give (2,6) type VEVs. A $<2,1,1;6,1>$breaks the gauge group to $SU_2(4) \times SU_3(2) \times SU(5) \times SU(6)$ with chiral fermions $2(1, 1; 5, 1) + (1, 2; 5, 1) + (2, 1; 5, 1) + (1, 1; 1, \bar{6}) + (2, 1; 1, \bar{6}) + (1, 2; 1, \bar{6}) + (1, 1, 1; \bar{5}, 6)$. There is insufficient fermion content for

a three family Pati-Salam model if we identify $SU_2(4) \times SU_3(2)$ with $SU_L(4) \times SU_R(2)$. Our only other choice is to get one of these $SU(2)$s from $SU(5) \times SU(6)$. For instance a $<2_2,5>$VEV breaks the gauge group to $SU_3(2) \times SU(4) \times SU(6)$ but the remaing chiral fermions are $4(1,4,1) + (2,4,1)$ $+3(1,1,\bar{6}) + (2,1,\bar{6}) + (1,2;1,\bar{6}) + (1,1,6) + (1,\bar{4},6)$. We can not identify $SU(4)$ with $SU_{PS}(4)$, so this group can only be in $SU(6)$. Breaking $SU(6)$ with an adjoint to $SU(2) \times SU(4)$ leaves us with $SU(2) \times SU(4) \times SU(2) \times SU(4)$ fermions that are again insufficient for a three family Pati-Salam model.

24/7 with $\mathbf{4} = (\mathbf{1}\alpha, \mathbf{1}', \mathbf{2}\alpha)$ for $N = 2$

This model, the only successful one in the present broad search, has been discussed in detail in [21] but for completeness we repeat the derivation here.

The original gauge group at the conformality scale is $SU(4)^3 \times SU(2)^{12}$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

If we break the three $SU(4)$s to a single diagonal $SU(4)$ subgroup, chirality is lost. To avoid this we break $SU(4)_1$ completely and then break $SU(4)_\alpha \times SU(4)_{\alpha^2}$ to its diagonal subgroup $SU(4)_D$. The appropriate VEVs are available as $[(4_1, 2_b\alpha^k) + h.c.]$ with b arbitrary but $k = 1$ or $k = 2$. The second step requires an $SU(4)_D$ singlet VEV from $(\bar{4}_\alpha, 4_{\alpha^2})$ and/or $(4_\alpha, \bar{4}_{\alpha^2})$. Once a choice is made for b (we take $b = 4$), the remaining fermions are, in an intuitive notation,:

$$\sum_{\alpha=1}^{\alpha=3} [(2_\alpha\alpha, 1, 4_D) + (1, 2_\alpha\alpha^{-1}, \bar{4}_D)] \tag{25}$$

which has the same content as a three family Pati-Salam model, though with a separate $SU(2)_L \times SU(2)_R$ per family.

To further reduce the symmetry we must arrange to break to a single $SU(2)_L$ and a single $SU(2)_R$. This is achieved by modifying step one where $SU(4)_1$ was broken. Consider the block diagonal decomposition of $SU(4)_1$ into $SU(2)_{1L} \times SU(2)_{1R}$. The representations $(2_\alpha\alpha, 4_1)$ and $(2_\alpha\alpha^{-1}, 4_1)$ decompose as $(2_\alpha\alpha, 4_1) \rightarrow (2_\alpha\alpha, 2, 1) + (2_\alpha\alpha, 1, 2)$ and $(2_\alpha\alpha^{-1}, 4_1) \rightarrow (2_\alpha\alpha^{-1}, 2, 1) + (2_\alpha^{-1}, 1, 2)$. Now if we give VEVs of equal magnitude to the $(2_a\alpha, 2, 1), a = 1, 2, 3$ and equal magnitude VEVs to the $(2_a\alpha^{-1}, 1, 2), a = 1, 2, 3$, we break $SU(2)_{1L} \times \Pi_{a=1}^{a=3} SU(2_a\alpha)$ to a single $SU(2)_L$ and we break $SU(2)_{1R} \times \Pi_{a=1}^{a=3} SU(2_a\alpha^{-1})$ to a single $SU(2)_R$. Finally, VEVs for $(2_4\alpha, 2, 1)$ and $(2_4\alpha, 1, 2)$ as well as $(2_4\alpha^{-1}, 2, 1)$ and $(2_4\alpha^{-1}, 1, 2)$ ensure that both $SU(2_4\alpha)$ and $SU(2_4\alpha^{-1})$ are broken and that only three families remain chiral. The final set of chiral fermions is then $3[(2, 1, 4) + (1, 2, \bar{4})]$ with gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_D$.

To achieve the final reduction to the standard model, an adjoint VEV from $(\bar{4}_\alpha, 4_{\alpha^2})$ and/or $(4_\alpha, \bar{4}_{\alpha^2})$ is used to break $SU(4)_D$ to $SU(3) \times U(1)$, and a right-handed doublet is used to break $SU(2)_R$.

<u>24/9 with $\mathbf{4} = (\mathbf{1}_1\alpha, \mathbf{1}_2\alpha^3, \mathbf{2}\alpha^2)$ for $N = 2$</u>

The original gauge group at the conformality scale is $SU(4)^4 \times SU(2)^8$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_2} = SU(2)_{1_1\alpha} = SU(2)_{1_2\alpha} = SU(2)_L$; $SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^2} = SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^3} = SU(2)_R$; while, for example, $SU(4)_2 = SU(4)_{2\alpha} = \bar{\mathbf{4}}$ of $SU(4)$; $SU(4)_{2\alpha^2} = SU(4)_{2\alpha^3} = \mathbf{4}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4} = (\mathbf{1}_1\alpha, \mathbf{1}_2\alpha, \mathbf{2}\alpha)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^4 \times SU(2)^8$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_2} = SU(2)_{1_1\alpha} = SU(2)_{1_2\alpha} = SU(2)_L$; $SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^2} = SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^3} = SU(2)_R$; while, for example, $SU(4)_2 = SU(4)_{2\alpha^3} = \bar{\mathbf{4}}$ of $SU(4)$; $SU(4)_{2\alpha} = SU(4)_{2\alpha^2} = \mathbf{4}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4} = (\mathbf{1}_1\alpha^2, \mathbf{1}_2, \mathbf{2}\alpha)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^4 \times SU(2)^8$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_2} = SU(2)_{1_1\alpha} = SU(2)_{1_2\alpha} = SU(2)_L$; $SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^2} = SU(2)_{1_1\alpha^3} = SU(2)_{1_2\alpha^3} = SU(2)_R$; while, for example, $SU(4)_2 = SU(4)_{2\alpha^3} = \bar{\mathbf{4}}$ of $SU(4)$; $SU(4)_{2\alpha} = SU(4)_{2\alpha^2} = \mathbf{4}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/9 with $\mathbf{4} = (\mathbf{2}\alpha, \mathbf{2}\alpha)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^4 \times SU(2)^8$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_1\alpha} = SU(2)_{1_1\alpha^2} = SU(2)_{1_1\alpha^3} = SU(2)_L$; $SU(2)_{1_2\alpha} = SU(2)_{1_2\alpha} = SU(2)_{1_2\alpha^2} = SU(2)_{1_2\alpha^3} = SU(2)_R$; while, for example, $SU(4)_{2\alpha} = SU(4)_{2\alpha^3} = \mathbf{4}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups, and $SU(4)_2$ and $SU(4)_{2\alpha^2}$ are broken.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/13 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)$ for $N = 2$

The original gauge group at the conformality scale is $SU(6) \times SU(4)^3 \times SU(2)^3$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

According to the analysis in Section IV this orbifold permits only two chiral families and is therefore not of phenomenological interest.

24/14 with $\mathbf{4} = (\mathbf{2}_1, \mathbf{2}_2)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^4 \times SU(2)^8$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_2} = SU(2)_{1_5} = SU(2)_{1_6} = SU(2)_L$; $SU(2)_{1_3} = SU(2)_{1_4} = SU(2)_{1_5} = SU(2)_{1_6} = SU(2)_R$; while, for example, $SU(4)_{2_2} = SU(4)_{2_3} = \mathbf{4}$

of $SU(4)$; $SU(4)_{2_1} = SU(4)_{2_4} = \bar{\mathbf{4}}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/15 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{1}_3, \mathbf{2}_3)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^5 \times SU(2)^4$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_3} = SU(2)_L$; $SU(2)_{1_2} = SU(2)_{1_4} = SU(2)_R$; while, for example, $SU(4)_{2_3} = SU(4)_{2_4} = \mathbf{4}$ of $SU(4)$, where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

24/15 with $\mathbf{4} = (\mathbf{2}_3, \mathbf{2}_5)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^5 \times SU(2)^4$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

According to the analysis in Section IV this orbifold permits only two chiral families and is hence not phenomenologically interesting.

24/15 with $\mathbf{4} = (\mathbf{2}_3, \mathbf{2}_3)$ for $N = 2$

The original gauge group at the conformality scale is $SU(4)^5 \times SU(2)^4$ with chiral fermions as given in Section IV and complex scalars as stated in Section V above.

Achievement of chiral families under the Pati-Salam subgroup $SU(4) \times SU(2)_L \times SU(2)_R$ requires the identifications $SU(2)_{1_1} = SU(2)_{1_3} = SU(2)_L$; $SU(2)_{1_2} = SU(2)_{1_4} = SU(2)_R$; while, for example, $SU(4)_{2_3} = SU(4)_{2_4} = \mathbf{4}$ of $SU(4)$ where by this simplified notation we imply diagonal subgroups.

But the scalars tabulated for this case in Section V are insufficient to allow this pattern of spontaneous symmetry breaking, and hence no interesting model emerges.

<u>27/4 with $\mathbf{4} = (\mathbf{1}_2, \mathbf{3}_1)$ with $N = 2$.</u>

Here $\mathbf{6} = (\mathbf{3}_1, \mathbf{3}_2)$ and the chiral fermions are given by Equation 29 and all scalars are of type $(2_i, \bar{6}_1), (2_i, 6_2)$ or $(6_1, \bar{6}_2)$ for $i = 1, 2, ..., 9$.. A VEV for $(6_1, \bar{6}_2) + h.c.$ scalar breaks $SU_1(6) \times SU_2(6)$ to $SU_D(6)$, and the model becomes vectorlike. Hence we must break only with (2,6) type scalars if there is any hope of a viable model. We give VEVs to $(2_i, 6_1)$ scalars for $i = 1, 2, ..., 5$ to break $SU_1(6)$ completely, and VEVs to $(2_j, 6_2)$ for $j = 6, 7$ to break $SU_2(6)$ to $SU(4)$. Then the remaining unbroken gauge group is $SU_8(2) \times SU_9(2) \times SU(4)$ with fermions $(2, 1, 4) + (1, 2, 4) + 4(1, 1, \bar{4})$, which are chiral but not of the correct form.

A more successful variation is obtained with $(2_i, 6_1)$ scalars VEVs for $i = 1, 2, 3$ and 4 to break the gauge group to $SU_5(2) \times SU_6(2) \times SU_7(2) \times SU_8(2) \times SU_9(2) \times SU'(2) \times SU(6)$ and thenVEVs for $(2_5, 6_2)$ and $(2_6, 6_2)$ to break to $SU_7(2) \times SU_8(2) \times SU_9(2) \times SU'(2) \times SU(4)$ which has chiral fermions (2,1,1,1,4)+(1,2,1,1,4)+(1,1,2,1,4)+3(1,1,1,2,$\bar{4}$). If we could break $SU_7(2) \times SU_8(2) \times SU_9(2)$ to a diagonal $SU(2)$ subgroup, we would have a three-family Pati-Salam model. However, the scalars to accomplish this are not in the spectrum. If we could give VEVs to $(2_i, 6_1)$ scalars for $i = 7, 8, 9$ to break $SU_7(2) \times SU_8(2) \times SU_9(2)$ to a $U_Y(1)$ without disturbing the $SU'(2)$ subgroup of $SU_1(6)$, and a further $(2_j, 6_2)$ VEV, say

$(2_1, 6_2)$, to break $SU(4)$ to $SU_C(3)$, then we would have a true three family standard (i.e., $U_Y(1) \times SU_{EW}(2) \times$ $SU_C(3)$)model upon identifying $SU'(2)$ with $SU_{EW}(2)$.

<u>30/2 with$\mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1_2}, \mathbf{2}\alpha)$ and $N = 2$</u>.

Here $\mathbf{6} = (\mathbf{1}_2\alpha, \mathbf{1}_2\alpha^{\mathbf{2}}, \mathbf{2}\alpha, \mathbf{2}\alpha^{\mathbf{2}})$, and the gauge group is $SU^6(2) \times SU^6(4)$. This group has chiral fermions:

$(2, 1, 1, 1, 1, 1; 1, 1, 4, 1, 1, 1) + (1, 2, 1, 1, 1, 1; 1, 1, 4, 1, 1, 1)$
$+ (1, 1, 2, 1, 1, 1; \bar{4}, 1, 1, 1, 1, 1) + (1, 1, 1, 2, 1, 1; \bar{4}, 1, 1, 1, 1, 1)$
$+(1, 1, 1, 1, 1, 1; 1, \bar{4}, 4, 1, 1, 1) + (1, 1, 1, 1, 1, 1; \bar{4}, 1, 4, 1, 1, 1)$
$+ 2(1, 1, 1, 1, 1, 1; 1, \bar{4}, 1, 4, 1, 1) + (1, 1, 1, 1, 1, 1;\ \bar{4}, 1, 1, 4, 1, 1)$
$+(1, 1, 2, 1, 1, 1; 1, 1, 1, 1, 4, 1) + (1, 1, 1, 2, 1, 1; 1, 1, 1, 1, 4, 1)$
$+ (1, 1, 1, 1, 2, 1; 1, 1, \bar{4}, 1, 1, 1) + (1, 1, 1, 1, 1, 2; 1, 1, \bar{4}, 1, 1, 1)$
$+(1, 1, 1, 1, 1, 1; 1, 1, 1, \bar{4}, 4, 1) + (1, 1, 1, 1, 1, 1; 1, 1, \bar{4}, 1, 4, 1)$
$+ 2(1, 1, 1, 1, 1, 1; 1, 1, 1, \bar{4}, 1, 4) + (1, 1, 1, 1, 1, 1; 1, 1, \bar{4}, 1, 1, 4)$
$+(1, 1, 1, 1, 2, 1; 4, 1, 1, 1, 1, 1) + (1, 1, 1, 1, 1, 2; 4, 1, 1, 1, 1, 1)$
$+ (2, 1, 1, 1, 1, 1; 1, 1, 1, 1, \bar{4}, 1) + (1, 2, 1, 1, 1, 1; 1, 1, 1, 1, \bar{4}, 1)$ $+(1, 1, 1, 1, 1, 1; 4, 1, 1, 1, \bar{4}, 1) +$ $(1, 1, 1, 1, 1, 1; 4, 1, 1, 1, 1, \bar{4})$
$+ 2(1, 1, 1, 1, 1, 1; 1, 4, 1, 1, 1, \bar{4}) + (1, 1, 1, 1, 1, 1; 1, 4, 1, 1, \bar{4}, 1)$

The spontaneous symmetry breaking analysis for this model is quite unwieldy, but for the most part can be carried out systematically. For example, breaking with $(1, 1, 1, 1, 1, 1; 1, \bar{4}, 1, 4, 1, 1)$, $(1, 1, 1, 1, 1, 1;$ $bar 4, 1, 1, 4, 1, 1)$, $(1, 1, 1, 1, 1, 1; 4, 1, 1, 1, \bar{4}, 1)$ and $(1, 1, 1, 1, 1, 1; 1, 1, \bar{4}, 1, 1, 4)$ VEVs reduces $SU^6(4)$ to $SU_1(4) \times SU_D(4)$, with fermions remaining chiral in representations:

$(2, 1, 1, 1, 1, 1; 1, 4) + (1, 2, 1, 1, 1, 1; 1, 4) + (1, 1, 2, 1, 1, 1; \bar{4}, 1) + (1, 1, 1, 2, 1, 1; \bar{4}, 1)$

$+(1,1,2,1,1,1;1,4)+(1,1,1,2,1,1;1,4)+(1,1,1,1,2,1;1,\bar{4})+(1,1,1,1,1,2;1,\bar{4})$

$+(1,1,1,1,2,1;4,1)+(1,1,1,1,1,2;4,1)+(2,1,1,1,1,1;1,\bar{4})+(1,2,1,1,1,1;1,\bar{4})$.

Now (1,1,1,1,2,1;4,1) and (1,1,1,1,1,2;4,1) VEVs break $SU_5(2) \times SU_6(2) \times SU_1(4)$ to $SU'(2)$ with fermions remaining chiral in the representations:

$(2,1,1,1;4)+(1,2,1,1;4)+(1,1,2,1;4)+(1,1,1,2;4)$

$+\,2(1,1,1,1;\bar{4})+2(1,1,1,1;\bar{4})+(2,1,1,1;\bar{4})+(1,2,1,1;\bar{4})$

which is already insufficient to provide three normal families. Other analyses of spontaneous symmetry breaking toward constructing a Pati-Salam model starting with this 30/2 model are similarly unsucessful.

An alternative is to seek a trinification model. To this end, consider only the $SU^6(4)$ fermion sector

$+(1,\bar{4},4,1,1,1)+(\bar{4},1,4,1,1,1)+2(1,\bar{4},1,4,1,1)$

$+\,(\bar{4},1,1,4,1,1)+(1,1,1,\bar{4},4,1)+(1,1,\bar{4},1,4,1)$

$+2(1,1,1,\bar{4},1,4)+(1,1,\bar{4},1,1,4)+(4,1,1,1,\bar{4},1)$

$+\,2(4,1,1,1,1,\bar{4})+2(1,4,1,1,1,\bar{4})+(1,4,1,1,\bar{4},1)$

Identifying $SU_1(4)$ with $SU_2(4)$, $SU_3(4)$ with $SU_4(4)$ and $SU_5(4)$ with $SU_6(4)$ would lead to five families of the form $5[(\bar{4},4,1)+(1,\bar{4},4)+(4,1,\bar{4})]$, however there are no scalars of the type needed to carry this out.

This analysis is not exhaustive and there may be models where $SU_L(2)$, $SU_R(2)$ or both are contained in $SU^6(4)$. Since we are starting with a group of rank 24, and seek the standard model of rank 4 or a unified model thereof of rank 5 or 6, and since there are 66 Higgs representations in the theory, the spontaneous symmetry breaking possibilities are rather complex. The $N=3$ case is obviously even more complicated, with initial rank 42, and one could try to automate the search for phenomenological models, although we have not attempted to do so.

<u>30/3 with $\mathbf{4} = (\mathbf{1_1}\alpha, \mathbf{1_2}, \mathbf{2}\alpha^{\mathbf{2}})$ and $N = 2$.</u> We now have $\mathbf{6} = (\mathbf{1}_2\alpha, \mathbf{1}_2\alpha^{\mathbf{4}}, \mathbf{2}\alpha^{\mathbf{3}}, \mathbf{2}\alpha^{\mathbf{2}})$ where $\alpha^{\mathbf{5}} = \mathbf{1}$.

The chiral $SU^{10}(2) \times SU^5(4)$fermions are

$(1^{10};\bar{4}, 4, 1, 1, 1)+(1^{10};\bar{4}, 1, 4, 1, 1)+(1^4, 2, 1^5;\bar{4}, 1, 1, 1, 1)+(1^5, 2, 1^4;\bar{4}, 1, 1, 1, 1)$
$+(1^{10};1,\bar{4}, 4, 1, 1)+(1^{10};1,\bar{4}, 1, 4, 1)+(1^6, 2, 1^3;1, \bar{4}, 1, 1, 1)+(1^7, 2, 1^2;1, \bar{4}, 1, 1, 1)$
$+(1^{10};1, 1, \bar{4}, 4, 1)+(1^{10};1,1,\bar{4}, 1, 4)+(1^8, 2, 1^1;\ 1, 1, \bar{4}, 1, 1)+(1^9, 2;1, 1, \bar{4}, 1, 1)$
$+(1^{10};1, 1, 1, \bar{4}, 4)+(1^{10};4, 1, 1, \bar{4}, 1)+(2,1^9;1, 1, 1, \bar{4}, 1)+(1^1, 2, 1^8;1, 1, 1, \bar{4}, 1)$
$+(1^{10};4, 1, 1, 1, \bar{4})+(1^{10};1, 4, 1, 1, \bar{4})+(1^2, 2, 1^7;\ 1, 1, 1, 1, \bar{4})+(1^3, 2, 1^6;1, 1, 1, 1, \bar{4})$ Consider the

bifundamentals only. VEVs for $(1, 1, 1, \bar{4}, 4)$ and $(1,\bar{4}, 4, 1, 1)$ scalars reduce the chiral fermion sector to $2[(\bar{4}, 4, 1) + (1, \bar{4}, 4) + (4, 1, \bar{4})]$ which provides at most a two family model.

If instead we try to construct a Pati-Salam model, and note that there are 20 (2;4) type fermions, and that we need six appropriate ones of these for three families, we must take care in the spontaneous symmetry breaking to preserve this much chirality. If we (i), break $SU_2(4)\times$ $SU_4(4) \times SU_5(4)$ completely and (ii) $SU_1(4) \times SU_3(4)$ to $SU_{PS}(4)$ while (iii) equating $SU_5(2)$, $SU_6(2)$, $SU_9(2)$ and (iv) equating $SU_{10}(2)$ with $SU_L(2)$, and $SU_1(2)$, $SU_2(2)$, $SU_7(2)$ and $SU_8(2)$ with $SU_R(2)$, and (v) breaking $SU_3(2) \times SU_4(2)$ completely, we would be left with a 4 family Pati–Salam Fmodel. Can we do this? (ii) is accomplished with (a) $(1^{10};\bar{4}, 1, 4, 1, 1)$, then (i) requires (b) $(1^{10};1,\bar{4}, 1, 4, 1)$ and (c) $(1^{10};1,\bar{4}, 1, 1, 4)$to get a $SU_D(4)$. Breaking this to nothing, assuming VEVs (a) and (b) allow no freedom to rotate the (c) VEV to diagonal form. Now, at this point, we are stymied, as there are insufficient $(2_i,2_j)$ representations of $SU_i(2)\times$ $SU_j(2)$ to accomplish (v).

Finally, one can imagine that there exist models with either $SU_L(2)$ or $SU_R(2)$ or both coming from $SU^5(4)$, but we see not obvious way to cary this out, while on the other hand since there are 60 Higgs representations we are unable to categorically eliminate this possibility.

VII. SUMMARY

We have shown how AdS/CFT duality leads to a large class of models which can provide interesting extensions of the standard model of particle phenomenology. The naturally occurring $\mathcal{N} = 4$ extended supersymmetry was completely broken to $\mathcal{N} = 0$ by choice of orbifolds S^5/Γ such that $\Gamma \not\subset SU(3)$.

In the present work, we studied systematically all such non-abelian Γ with order $g \leq 31$. We have seen how chiral fermions require that the embedding of Γ be neither real nor pseudoreal. This reduces dramatically the number of possibilities to obtain chiral fermions. Nevertheless, many candidates for models which contain the chiral fermions of the three-family standard model were found.

However, the requirement that the spontaneous symmetry breaking down to the correct gauge symmetry of the standard model be permitted by the prescribed scalar representations eliminates most of the surviving models. We found only one allowed model based on the $\Gamma = 24/7$ orbifold. We had initially expected to find more examples in our search. The moral for model-building is interesting. Without the rigid framework of string duality the scalar sector would be arbitrarily chosen to permit the required spontaneous symmetry breaking. This is the normal practice in the standard model, in grand unification, in supersymmetry and so on. With string duality, the scalar sector is prescribed by the construction and only in one very special case does it permit the required symmetry breaking.

This leads us to give more credence to the $\Gamma = 24/7$ example that does work and to encourage its further study to check whether it can have any connection to the real world.

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APPENDIX: MULTIPLICATION TABLES FOR NON-ABELIAN GROUPS WITH $G \leq 31$

The group $D_3 = S_3$

$\otimes$	1	$1'$	2
1	1	$1'$	2
$1'$	$1'$	1	2
2	2	2	$1+1'+2$

The group D_4, 8/4

$\otimes$	1_1	1_2	1_3	1_4	2
1_1	1_1	1_2	1_3	1_4	2
1_2	1_2	1_1	1_4	1_3	2
1_3	1_3	1_4	1_1	1_2	2
1_4	1_4	1_3	1_2	1_1	2
2	2	2	2	2	$1_1+1_2+1_3+1_4$

The group Q, 8/5

$\otimes$	1_1	1_2	1_3	1_4	2
1_1	1_1	1_2	1_3	1_4	2
1_2	1_2	1_1	1_4	1_3	2
1_3	1_3	1_4	1_1	1_2	2
1_4	1_4	1_3	1_2	1_1	2
2	2	2	2	2	$1_1+1_2+1_3+1_4$

The group D_5, 10/2

$\otimes$	1	$1'$	2	$2'$
1	1	$1'$	$1''$	$2'$
$1'$	$1'$	1	1	$2'$
2	2	2	$1+1'+2'$	$2+2'$
$2'$	$2'$	$2'$	$2+2'$	$1+1'+2$

The group T, 12/4

$\otimes$	1	$1'$	$1''$	3
1	1	$1'$	$1''$	3
$1'$	$1'$	$1''$	1	3
$1''$	$1''$	1	$1'$	3
3	3	3	3	$1+1'+1''+3+3$

The group $D_6, 12/3$

$\otimes$	1_1	1_2	1_3	1_4	2	$2'$
1_1	1_1	1_2	1_3	1_4	2	$2'$
1_2	1_2	1_1	1_4	1_3	$2'$	2
1_3	1_3	1_4	1_1	1_2	$2'$	2
1_4	1_4	1_3	1_2	1_1	2	$2'$
2	2	$2'$	$2'$	2	$1_1+1_4+2'$	1_2+1_3+2
$2'$	$2'$	2	2	$2'$	1_2+1_3+2	$1_1+1_4+2'$

The group $Q_6, 12/5$

$\otimes$	1_1	1_2	1_3	1_4	2	$2'$
1_1	1_1	1_2	1_3	1_4	2	$2'$
1_2	1_2	1_3	1_4	1_1	$2'$	2
1_3	1_3	1_4	1_1	1_2	2	$2'$
1_4	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	$1_1 + 1_3 + 2'$	$1_2 + 1_4 + 2$
$2'$	$2'$	2	$2'$	2	$1_2 + 1_4 + 2$	$1_1 + 1_2 + 2'$

The group $D_7, 14/2$

$\otimes$	1	$1'$	2_1	2_2	2_3
1	1_1	$1'$	2_1	2_2	2_3
$1'$	$1'$	1	2_1	2_2	2_3
2_1	2_1	2_1	$1 + 1' + 2_2$	$2_1 + 2_3$	$2_2 + 2_3$
2_2	2_2	2_2	$2_1 + 2_3$	$1 + 1' + 2_3$	$2_1 + 2_2$
2_3	2_3	2_3	$2_2 + 2_3$	$2_1 + 2_2$	$1 + 1' + 2_1$

The group $(Z_4 \times Z_2)\widetilde{\times} Z_2, 16/8$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_1	1_4	1_3	1_6	1_5	1_8	1_7	2	$2'$
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_3	1_2	1_1	1_8	1_7	1_6	1_5	2	$2'$
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	$2'$	2
1_6	1_6	1_5	1_8	1_7	1_2	1_1	1_4	1_3	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	$2'$	2
1_8	1_8	1_7	1_6	1_5	1_4	1_3	1_2	1_1	$2'$	2
2	2	2	2	2	$2'$	$2'$	$2'$	$2'$	$1_5 + 1_6 + 1_7 + 1_8$	$1_1 + 1_2 + 1_3 + 1_4$
$2'$	$2'$	$2'$	$2'$	$2'$	2	2	2	2	$1_1 + 1_2 + 1_3 + 1_4$	$1_5 + 1_6 + 1_7 + 1_8$

The group $Z_4 \tilde{\times} Z_4, 16/10$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_3	1_4	1_1	1_6	1_7	1_8	1_5	$2'$	2
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_1	1_2	1_3	1_8	1_5	1_6	1_7	$2'$	2
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2	$2'$
1_6	1_6	1_7	1_8	1_5	1_2	1_3	1_4	1_1	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	2	$2'$
1_8	1_8	1_5	1_6	1_7	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	2	$2'$	2	$2'$	$1_1 + 1_3 + 1_5 + 1_7$	$1_2 + 1_4 + 1_6 + 1_8$
$2'$	$2'$	2	$2'$	2	$2'$	2	$2'$	2	$1_2 + 1_4 + 1_6 + 1_8$	$1_1 + 1_3 + 1_5 + 1_7$

The group $Z_8 \widetilde{\times} Z_2, 16/11$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2	$2'$
1_2	1_2	1_3	1_4	1_1	1_6	1_7	1_8	1_5	$2'$	2
1_3	1_3	1_4	1_1	1_2	1_7	1_8	1_5	1_6	2	$2'$
1_4	1_4	1_1	1_2	1_3	1_8	1_5	1_6	1_7	$2'$	2
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2	$2'$
1_6	1_6	1_7	1_8	1_5	1_2	1_3	1_4	1_1	$2'$	2
1_7	1_7	1_8	1_5	1_6	1_3	1_4	1_1	1_2	2	$2'$
1_8	1_8	1_5	1_6	1_7	1_4	1_1	1_2	1_3	$2'$	2
2	2	$2'$	2	$2'$	2	$2'$	2	$2'$	$1_2 + 1_4 + 1_6 + 1_8$	$1_1 + 1_3 + 1_5 + 1_7$
$2'$	$2'$	2	$2'$	2	$2'$	2	$2'$	2	$1_1 + 1_3 + 1_5 + 1_7$	$1_2 + 1_4 + 1_6 + 1_8$

The group D_8, $(Z_8 \tilde{\times} Z_2)'$, 16/12 ($Q_8$, 16/14, has the same table.)

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_2	1_2	1_1	1_4	1_3	2_3	2_2	2_1
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3
1_4	1_4	1_3	1_2	1_1	2_3	2_2	2_1
2_1	2_1	2_3	2_1	2_3	$1_1 + 1_3 + 2_2$	$2_1 + 2_3$	$1_2 + 1_4 + 2_2$
2_2	2_2	2_2	2_2	2_2	$2_1 + 2_3$	$1_1 + 1_2 + 1_3 + 1_4$	$2_1 + 2_3$
2_3	2_3	2_1	2_3	2_1	$1_2 + 1_4 + 2_2$	$2_1 + 2_3$	$1_1 + 1_3 + 2_2$

The group $(Z_8 \tilde{\times} Z_2)''$, 16/13

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3
1_2	1_2	1_1	1_4	1_3	2_3	2_2	2_1
1_3	1_3	1_4	1_1	1_2	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_1	2_2	2_3
2_1	2_1	2_3	2_3	2_1	$1_2 + 1_3 + 2_2$	$2_1 + 2_3$	$1_1 + 1_4 + 2_2$
2_2	2_2	2_2	2_2	2_2	$2_1 + 2_3$	$1_1 + 1_2 + 1_3 + 1_4$	$2_1 + 2_3$
2_3	2_3	2_1	2_1	2_3	$1_1 + 1_4 + 2_2$	$2_1 + 2_3$	$1_2 + 1_3 + 2_2$

The group $D_9, 18/4$

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4
1	1	$1'$	2_1	2_2	2_3	2_4
$1'$	$1'$	1	2_1	2_2	2_3	2_4
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_2+2_4	2_3+2_4
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_4	2_2+2_3
2_3	2_3	2_3	2_2+2_4	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_4	2_4	2_4	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

The group $(Z_3 \times Z_3)\widetilde{\times} Z_2, 18/5$

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4
1	1	$1'$	2_1	2_2	2_3	2_4
$1'$	$1'$	1	2_1	2_2	2_3	2_4
2_1	2_1	2_1	$1+1'+2_1$	2_3+2_4	2_2+2_4	2_2+2_3
2_2	2_2	2_2	2_3+2_4	$1+1'+2_2$	2_1+2_4	2_1+2_3
2_3	2_3	2_3	2_2+2_4	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_4	2_4	2_4	2_2+2_3	2_1+2_3	2_1+2_2	$1+1'+2_4$

The group D_{10}, 20/3

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_2	1_2	1_1	1_4	1_3	2_4	2_3	2_1	2_1
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3	2_4
1_4	1_4	1_3	1_2	1_1	2_4	2_3	2_1	2_1
2_1	2_1	2_4	2_1	2_4	$1_1 + 1_3 \ + 2_2$	$2_1 + 2_3$	$2_2 + 2_4$	$1_2 + 1_4 + 2_3$
2_2	2_2	2_3	2_2	2_3	$2_1 + 2_3$	$1_1 + 1_3 \ + 2_4$	$2_1 + 2_3$	$2_2 + 2_4$
2_3	2_3	2_2	2_3	2_2	$2_2 + 2_4$	$2_1 + 2_3$	$1_1 + 1_3 \ + 2_4$	$2_1 + 2_3$
2_4	2_4	2_1	2_4	2_1	$1_2 + 1_4 + 2_3$	$2_2 + 2_4$	$2_1 + 2_3$	$1_1 + 1_3 \ + 2_2$

The group $Z_5 \widetilde{\times} Z_4$, 20/5

$\otimes$	1_1	1_2	1_3	1_4	4
1_1	1_1	1_2	1_3	1_4	4
1_2	1_2	1_3	1_4	1_1	4
1_3	1_3	1_4	1_1	1_2	4
1_4	1_4	1_1	1_2	1_3	4
4	4	4	4	4	$1_1 + 1_2 + 1_3 + 1_4 + 3 \times 4$

The group $Z_7 \tilde{\times} Z_3$, 21/2

$\otimes$	1_1	1_2	1_3	3_1	3_2
1_1	1_1	1_2	1_3	3_1	3_2
1_2	1_2	1_3	1_1	3_1	3_2
1_3	1_3	1_1	1_2	3_1	3_2
3_1	3_1	3_1	3_1	$3_1+3_2+3_2$	$1_1+1_2+1_3+3_1+3_2$
3_2	3_2	3_2	3_2	$1_1+1_2+1_3+3_1+3_2$	$3_1+3_1+3_2$

The group $D_{11}, 22/2$

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5
1	1	$1'$	2_1	2_2	2_3	2_4	2_5
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_5
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_5	2_3+2_4
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_4	2_4	2_4	2_3+2_5	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_5	2_5	2_5	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

The group D_{12}, 24/10

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_2	1_2	1_1	1_4	1_3	2_1	2_2	2_3	2_4	2_5
1_3	1_3	1_4	1_1	1_2	2_5	2_4	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_5	2_4	2_3	2_2	2_1
2_1	2_1	2_1	2_5	2_5	$1_1 + 1_2 \ + 2_2$	$2_1 + 2_3$	$2_2 + 2_4$	$2_3 + 2_5$	$2_3 + 2_5$
2_2	2_2	2_2	2_4	2_4	$2_1 + 2_3$	$1_1 + 1_2 \ + 2_4$	$2_1 + 2_5$	$1_3 \ + 1_4 + 2_4$	$2_3 + 2_5$
2_3	2_3	2_3	2_3	2_3	$2_2 + 2_4$	$2_1 + 2_5$	$1_1 + 1_2 \ + 1_3 \ + 1_4$	$2_1 + 2_5$	$2_2 + 2_4$
2_4	2_4	2_4	2_2	2_2	$2_3 + 2_5$	$1_3 \ + 1_4 + 2_4$	$2_1 + 2_5$	$1_1 + 1_2 \ + 2_4$	$2_1 + 2_3$
2_5	2_5	2_5	2_1	2_1	$2_3 + 2_5$	$2_3 + 2_5$	$2_2 + 2_4$	$2_1 + 2_3$	$1_1 + 1_2 \ + 2_2$

The group S_4, 24/12

$\otimes$	1	$1'$	2	3	$3'$
1	1	$1'$	2	3	$3'$
$1'$	$1'$	1	2	$3'$	3
2	2	2	$1 + 1' + 2$	$3 + 3'$	$3 + 3'$
3	3	$3'$	$3 + 3'$	$1 + 2 + 3 + 3'$	$1' + 2 + 3 + 3'$
$3'$	$3'$	3_2	$3 + 3'$	$1' + 2 + 3 + 3'$	$1 + 2 + 3 + 3'$

The group $SL_2(F_3)$, $Q \tilde{\times} Z_3, 24/13$

$\otimes$	1_1	1_2	1_3	2_1	2_2	2_3	3
1_1	1_1	1_2	1_3	2_1	2_2	2_3	3
1_2	1_2	1_3	1_1	2_2	2_3	2_1	3
1_3	1_3	1_1	1_2	2_3	2_1	2_2	3
2_1	2_1	2_2	2_3	$1+3$	$1'+3$	$1''+3$	$2_1+2_2+2_3$
2_2	2_2	2_3	2_1	$1'+3$	$1''+3$	$1+3$	$2_1+2_2+2_3$
2_3	2_3	2_1	2_2	$1''+3$	$1+3$	$1'+3$	$2_1+2_2+2_3$
3	3	3	3	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$2_1+2_2+2_3$	$1_1+1_2+1_3\ +3+3$

The group $Z_8 \tilde{\times} Z_3$, 24/14

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	2_1	2_2	2_3	2_4
1_2	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_1	2_2	2_3	2_4	2_1
1_3	1_3	1_4	1_5	1_6	1_7	1_8	1_1	1_2	2_3	2_4	2_1	2_2
1_4	1_4	1_5	1_6	1_7	1_8	1_1	1_2	1_3	2_4	2_1	2_2	2_3
1_5	1_5	1_6	1_7	1_8	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4
1_6	1_6	1_7	1_8	1_1	1_2	1_3	1_4	1_5	2_2	2_3	2_4	2_1
1_7	1_7	1_8	1_1	1_2	1_3	1_4	1_5	1_6	2_3	2_4	2_1	2_2
1_8	1_8	1_1	1_2	1_3	1_4	1_5	1_6	1_7	2_4	2_1	2_2	2_3
2_1	2_1	2_2	2_3	2_4	2_1	2_2	2_3	2_4	$1_1+1_5+2_1$	$1_2+1_6+2_2$	$1_3+1_7+2_3$	$1_4+1_8+2_4$
2_2	2_2	2_3	2_4	2_1	2_2	2_3	2_4	2_1	$1_2+1_6+2_2$	$1_3+1_7+2_3$	$1_4+1_8+2_4$	$1_1+1_5+2_1$
2_3	2_3	2_4	2_1	2_2	2_3	2_4	2_1	2_2	$1_3+1_7+2_3$	$1_4+1_8+2_4$	$1_1+1_5+2_1$	$1_2+1_6+2_2$
2_4	2_4	2_1	2_2	2_3	2_4	2_1	2_2	2_3	$1_4+1_8+2_4$	$1_1+1_5+2_1$	$1_2+1_6+2_2$	$1_3+1_7+2_3$

The group $D_4 \widetilde{\times} Z_3$, 24/15

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5
1_2	1_2	1_1	1_4	1_3	2_2	2_1	2_4	2_3	2_5
1_3	1_3	1_4	1_1	1_2	2_1	2_2	2_3	2_4	2_5
1_4	1_4	1_3	1_2	1_1	2_2	2_1	2_4	2_3	2_5
2_1	2_1	2_2	2_1	2_2	$1_1+1_3+2_1$	$1_2+1_4+2_2$	2_4+2_5	2_3+2_5	2_3+2_4
2_2	2_2	2_1	2_2	2_1	$1_2+1_4+2_2$	$1_1+1_3+2_1$	2_3+2_5	2_4+2_5	2_3+2_4
2_3	2_3	2_4	2_3	2_4	2_4+2_5	2_3+2_5	$1_2+1_4+2_1$	$1_1+1_3+2_2$	2_1+2_2
2_4	2_4	2_3	2_4	2_3	2_3+2_5	2_4+2_5	$1_1+1_3+2_2$	$1_2+1_4+2_1$	2_1+2_2
2_5	2_5	2_5	2_5	2_5	2_3+2_4	2_3+2_4	2_1+2_2	2_1+2_2	$1_1+1_2+1_3+1_4$

The group $D_{13}, 26/2$

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6
1	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5	2_6
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_6	2_5+2_6
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_6	2_3+2_6	2_4+2_5
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_6$	2_1+2_6	2_2+2_5	2_3+2_4
2_4	2_4	2_4	2_3+2_5	2_2+2_6	2_1+2_6	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_5	2_5	2_5	2_4+2_6	2_3+2_6	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_6	2_6	2_6	2_5+2_6	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

The group $(Z_3 \times Z_3)\widetilde{\times} Z_3, 27/4$

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_2	1_2	1_3	1_1	1_5	1_6	1_4	1_8	1_9	1_7	3_1	3_2
1_3	1_3	1_1	1_2	1_6	1_4	1_5	1_9	1_7	1_8	3_1	3_2
1_4	1_4	1_5	1_6	1_7	1_8	1_9	1_1	1_2	1_3	3_1	3_2
1_5	1_5	1_6	1_4	1_8	1_9	1_7	1_2	1_3	1_1	3_1	3_2
1_6	1_6	1_4	1_5	1_9	1_7	1_8	1_3	1_1	1_2	3_1	3_2
1_7	1_7	1_8	1_9	1_1	1_2	1_3	1_4	1_5	1_6	3_1	3_2
1_8	1_8	1_9	1_7	1_2	1_3	1_1	1_5	1_6	1_4	3_1	3_2
1_9	1_9	1_7	1_8	1_3	1_1	1_2	1_6	1_4	1_5	3_1	3_2
3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	$3{\times}3_2$	$\sum_{i=1}^{9} 1_i$
3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	$\sum_{i=1}^{9} 1_i$	$3{\times}3_1$

The group $Z_9 \widetilde{\times} Z_3, 27/5$ [Note this table is the same as for $(Z_3 \times Z_3)\widetilde{\times} Z_3$.]

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_1	1_1	1_2	1_3	1_4	1_5	1_6	1_7	1_8	1_9	3_1	3_2
1_2	1_2	1_3	1_1	1_5	1_6	1_4	1_8	1_9	1_7	3_1	3_2
1_3	1_3	1_1	1_2	1_6	1_4	1_5	1_9	1_7	1_8	3_1	3_2
1_4	1_4	1_5	1_6	1_7	1_8	1_9	1_1	1_2	1_3	3_1	3_2
1_5	1_5	1_6	1_4	1_8	1_9	1_7	1_2	1_3	1_1	3_1	3_2
1_6	1_6	1_4	1_5	1_9	1_7	1_8	1_3	1_1	1_2	3_1	3_2
1_7	1_7	1_8	1_9	1_1	1_2	1_3	1_4	1_5	1_6	3_1	3_2
1_8	1_8	1_9	1_7	1_2	1_3	1_1	1_5	1_6	1_4	3_1	3_2
1_9	1_9	1_7	1_8	1_3	1_1	1_2	1_6	1_4	1_5	3_1	3_2
3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	3_1	$3 \times 3_2$	$\sum_{i=1}^{9} 1_i$
3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	3_2	$\sum_{i=1}^{9} 1_i$	$3 \times 3_1$

The group D_{14}, 28/3

$\otimes$	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5	2_6
1_1	1_1	1_2	1_3	1_4	2_1	2_2	2_3	2_4	2_5	2_6
1_2	1_2	1_1	1_4	1_3	2_1	2_2	2_3	2_4	2_5	2_6
1_3	1_3	1_4	1_1	1_2	2_6	2_5	2_4	2_3	2_2	2_1
1_4	1_4	1_3	1_2	1_1	2_6	2_5	2_4	2_3	2_2	2_1
2_1	2_1	2_1	2_6	2_6	$1_1+1_2\ +2_2$	2_1+2_3	2_2+2_4	2_3+2_5	2_4+2_6	$1_3+1_4\ +2_5$
2_2	2_2	2_2	2_5	2_5	2_1+2_3	$1_1+1_2\ +2_4$	2_1+2_5	2_2+2_6	$1_3+1_4\ +2_3$	2_4+2_6
2_3	2_3	2_3	2_4	2_4	2_2+2_4	2_1+2_5	$1_1+1_2\ +2_6$	$1_3+1_4\ +2_1$	2_2+2_6	2_3+2_5
2_4	2_4	2_4	2_3	2_3	2_3+2_5	2_2+2_6	$1_3+1_4\ +2_1$	$1_1+1_2\ +2_6$	2_1+2_5	2_2+2_4
2_5	2_5	2_5	2_2	2_2	2_4+2_6	$1_3+1_4\ +2_3$	2_2+2_6	2_1+2_5	$1_1+1_2\ +2_4$	2_1+2_3
2_6	2_6	2_6	2_1	2_1	$1_3+1_4\ +2_5$	2_4+2_6	2_3+2_5	2_2+2_4	2_1+2_3	$1_1+1_2\ +2_2$

The group $D_5 \times Z_3$, 30/2

$\otimes$	1_1	1_2	1_3	1_4	1_5	1_6	2_1	2_2	2_3	2_4	2_5	2_6
1_1	1_1	1_2	1_3	1_4	1_5	1_6	2_1	2_2	2_3	2_4	2_5	2_6
1_2	1_2	1_3	1_4	1_5	1_6	1_1	2_5	2_6	2_1	2_2	2_3	2_4
1_3	1_3	1_4	1_5	1_6	1_1	1_2	2_3	2_4	2_5	2_6	2_1	2_2
1_4	1_4	1_5	1_6	1_1	1_2	1_3	2_1	2_2	2_3	2_4	2_5	2_6
1_5	1_5	1_6	1_1	1_2	1_3	1_4	2_5	2_6	2_1	2_2	2_3	2_4
1_6	1_6	1_1	1_2	1_3	1_4	1_5	2_3	2_4	2_5	2_6	2_1	2_2
2_1	2_1	2_5	2_3	2_1	2_5	2_3	$1_1 + 1_4 + 2_1$	$2_1 + 2_2$	$1_3 + 1_6 + 2_4$	$2_3 + 2_4$	$1_2 + 1_5 + 2_6$	$2_5 + 2_6$
2_2	2_2	2_6	2_4	2_2	2_6	2_4	$2_1 + 2_1$	$1_1 + 1_4 + 2_2$	$2_3 + 2_4$	$1_3 + 1_6 + 2_3$	$2_5 + 2_6$	$1_2 + 1_5 + 2_5$
2_3	2_3	2_1	2_5	2_3	2_1	2_5	$1_3 + 1_6 + 2_4$	$2_3 + 2_4$	$1_2 + 1_5 + 2_6$	$2_5 + 2_6$	$1_1 + 1_4 + 2_2$	$2_1 + 2_1$
2_4	2_4	2_2	2_6	2_4	2_2	2_6	$2_3 + 2_4$	$1_3 + 1_6 + 2_3$	$2_5 + 2_6$	$1_2 + 1_5 + 2_5$	$2_1 + 2_2$	$1_1 + 1_4 + 2_1$
2_5	2_5	2_3	2_1	2_5	2_3	2_1	$1_2 + 1_5 + 2_6$	$2_5 + 2_6$	$1_1 + 1_4 + 2_2$	$2_1 + 2_2$	$1_3 + 1_6 + 2_4$	$2_3 + 2_4$
2_6	2_6	2_4	2_2	2_6	2_4	2_2	$2_5 + 2_6$	$1_2 + 1_5 + 2_5$	$2_1 + 2_2$	$1_1 + 1_5 + 2_1$	$2_3 + 2_4$	$1_3 + 1_6 + 2_3$

The group $D_{15}, 30/4$

$\otimes$	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6	2_7
1	1	$1'$	2_1	2_2	2_3	2_4	2_5	2_6	2_7
$1'$	$1'$	1	2_1	2_2	2_3	2_4	2_5	2_6	2_7
2_1	2_1	2_1	$1+1'+2_2$	2_1+2_3	2_1+2_4	2_3+2_5	2_4+2_6	2_5+2_7	2_6+2_7
2_2	2_2	2_2	2_1+2_3	$1+1'+2_4$	2_1+2_5	2_2+2_6	2_3+2_7	2_4+2_7	2_5+2_6
2_3	2_3	2_3	2_1+2_4	2_1+2_5	$1+1'+2_6$	2_1+2_7	2_2+2_7	2_3+2_6	2_4+2_5
2_4	2_4	2_4	2_3+2_5	2_2+2_6	2_1+2_7	$1+1'+2_7$	2_1+2_6	2_2+2_5	2_3+2_4
2_5	2_5	2_5	2_4+2_6	2_3+2_7	2_2+2_7	2_1+2_6	$1+1'+2_5$	2_1+2_4	2_2+2_3
2_6	2_6	2_6	2_5+2_7	2_4+2_7	2_3+2_6	2_2+2_5	2_1+2_4	$1+1'+2_3$	2_1+2_2
2_7	2_7	2_7	2_6+2_7	2_5+2_6	2_4+2_5	2_3+2_4	2_2+2_3	2_1+2_2	$1+1'+2_1$

FIGURE CAPTION.

Quiver diagram for chiral fermions in 24/7 model.